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Slant path intensity distribution of focusing J_0-correlated Schell-model vortex beams in atmospheric turbulence

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Abstract

The intensity distribution of the J_0-correlated Schell-model (JSM) vortex beams focused by a lens and propagation in weak-to-strong turbulent atmosphere are investigated. It is shown that the beam spreading increases with the increase in topological charge n, the source coherent length $\alpha-1$, turbulent outer scale L_0 and propagation distance z. The center hollow depth of intensity distribution of the J_0-correlated Schell-model (JSM) vortex beams decrease with the increase of topological charge n, turbulent outer scale L_0 and propagation distance z or the decrease of the source coherent length $\alpha-1$.

Keywords: Atmospheric turbulence; J_0-correlated Schell beams; Vortex; Intensity distribution

1. Introduction

The propagation of laser beams through atmospheric turbulence has attracted much attention because of its important practical applications, such as in connection with remote sensing, imaging, and communication systems [1–10]. In recent years, much work has been carried out concerning the properties of optical beams that possess an intensity null along their propagation axis and hence a singular phase on that axis, due to their interesting characteristics and wide application [2,9–11].

The study of such 'vortex beams' (so called because the phase circulates about the central null, much like a fluid circulating a drain) has become an important subfield of the general study of phase singularities in optical fields, known now as singular optics. Such beams are of particular interest because they carry orbital angular momentum [12] and the orbital angular momentum may help vortex beams propagate through optical turbulence with less distortion than conventional Gaussian beams [13]. However, the studies undertaken so far have typically been limited to rather specialized circumstances, namely, interference-generated vortices [14] and partially coherent vortex beams [9,15], the topological charge properties of vortex beams propagating through atmospheric turbulence [11], as far as we know, however, there are no paper concerning the propagation of the focusing J_0-correlated Schell-model vortex beams in through turbulence.

In this paper, the intensity distribution of the J_0-correlated Schell-model vortex beams focused by a lens and propagation through slant path atmospheric turbulence have been studied with the theory of cross-spectral density and the quadratic approximation of the long-term wave structure function.

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2. Theoretical model

Let us first consider a JSM vortex beam is focused by a lens with the focal length f. We assumed the field $E^0(r,\omega)$ in the lens is a partially coherent Gaussian model field with optical vortex; i.e., the field may be written as [16,17]

$$E^0(r,\omega) = E_0 \exp\left(-\frac{r^2}{w_0^2}\right) \exp(\mathrm{i}m\phi) \exp(\mathrm{i}\omega t) \exp(\mathrm{i}\beta)$$
$$n = 0, 1, 2, 3, \ldots \quad (1)$$

where E_0 and w_0 are the constant amplitude and the beam width, ω is the angular frequency, m is the topological charge, and β is an arbitrary phase (as a spatially distributed random variable). We assume that the statistical distribution of β corresponds to a J_0-correlated Schell-model [18] $C(r_1, r_2) = J_0(\alpha|r_1 - r_2)$, where α^{-1} corresponds to the coherence length of incident JMS vortex beam. According to the theory of optical coherence in space frequency domain and using J_0-correlated Schell-model, the coherence properties of a partially coherence wave field may be described by the cross-spectral density

$$W^0(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle E^*(\mathbf{r}_1,\omega)E(\mathbf{r}_2,\omega)\rangle = E_0^2 \exp\left(-\frac{r_1^2+r_2^2}{w_0^2}\right) \times J_0(\alpha|\mathbf{r}_1 - \mathbf{r}_2|) \exp[-\mathrm{i}m(\phi_1 - \phi_2)] \quad (2)$$

where the angular brackets denote the ensemble average, taken over a statistical ensemble of monochromatic realizations, and the asterisk denotes the complex conjugate, $\mathbf{r}_1 \equiv (r_1, \phi_1)$ and $\mathbf{r}_2 \equiv (r_2, \phi_2)$ are the position vectors corresponding to two points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ in the $z = 0$ plane, and J_0 is the zeroth-order Bessel function of the first kind.

According to the generalized Huygens–Fresnel diffraction integral and in the paraxial approximation [19], the cross-spectral density through an optical system is given by

$$W(\mathbf{r}'_1,\mathbf{r}'_2,z) = \left(\frac{k}{2\pi z}\right)^2 \exp\left[-\frac{\mathrm{i}k}{2z}(r_1'^2 - r_2'^2)\right] \times \int\int \exp\left(-\frac{\mathrm{i}k}{2z}\left[\left(1-\frac{z}{f}\right)(r_1^2 - r_2^2) - 2(\mathbf{r}'_1\cdot\mathbf{r}_1 - \mathbf{r}'\cdot_2\mathbf{r}_2)\right]\right) \times \exp\left(-\frac{1}{2}D_\psi(\mathbf{r}'_1 - \mathbf{r}'_2, \mathbf{r}_1 - \mathbf{r}_2)\right) W^0(\mathbf{r}'_1,\mathbf{r}'_2,0)\,\mathrm{d}\mathbf{r}_2\,\mathrm{d}\mathbf{r}_2 \quad (3)$$

where $k = 2\pi/\lambda$ is the wave number, $D_\psi(\mathbf{r}'_1 - \mathbf{r}'_2, \mathbf{r}_1 - \mathbf{r}_2)$ is wave structure function. Based on the modified von Karman spectrum model of index-of-refraction fluctuation [10]

$$\phi_n(\kappa) = 0.033C_n^2 \exp\left(-\frac{\kappa^2}{\kappa_m^2}\right)[\kappa_0^2 + \kappa^2]^{-11/6}$$
$$\kappa_0 = \frac{2\pi}{L_0}, \quad \kappa_m = \frac{5.92}{l_0} \quad (4)$$

where L_0 is outer scale of turbulence and l_0 is inner scale of turbulence, and the quadratic approximation, we have the long-term wave structure function [10]

$$D_{\psi LT}(\mathbf{r}'_1 - \mathbf{r}'_2, \mathbf{r}_1 - \mathbf{r}_2) \approx 2\rho_0^{-2}[|\mathbf{r}'_1 - \mathbf{r}'_2|^2 + |\mathbf{r}_1 - \mathbf{r}_2|^2 + |\mathbf{r}'_1 - \mathbf{r}'_2||\mathbf{r}_1 - \mathbf{r}_2|] \quad (5)$$

here

$$\tilde{\rho}_0^2 = \rho_0^2[1 - 0.715\kappa_0^{1/3}]^{-1}$$
$$\rho_0^2 = \left[1.45k^2 \sec\theta \int_{z_0}^{z_1} C_n^2(z)\left(\frac{z_1 - z}{z_1 - z_0}\right)^{5/3} \mathrm{d}z\right]^{-6/5} \times [1 - 0.103(3/8)^{-5/6}\kappa_0^{1/3}]$$

is the spatial coherence radius of a spherical wave propagating in turbulence, $C_n^2(z)$ is the refractive index structure parameter). One of the most widely used models is the Hufnagel-Velly model described by [1]

$$C_n^2(z\cos\theta) = 0.00594(v/27)^2 \times (z\cos\theta \times 10^{-5})^{10} \exp(-z\cos\theta/1000) + 2.7\times 10^{-16}\exp(-z\cos\theta/1500) + C_n^2(0)\exp(-z\cos\theta/100) \quad (6)$$

here $v = 2.1\,\mathrm{m/s}$ is the root-mean-square wind speed and $C_n^2(0) = 1.7\times 10^{-14}\,\mathrm{m}^{-2/3}$ or $C_n^2(0) = 3\times 10^{-13}\,\mathrm{m}^{-2/3}$ is the refractive index structural characteristic of ground, θ is the zenith angle. The short-term wave structure function can be written as [10]

$$D_{\psi LT}(\mathbf{r}'_1 - \mathbf{r}'_2, \mathbf{r}_1 - \mathbf{r}_2) = 2[1 - (\rho_0/2w_0)^{1/3}]\tilde{\rho}_0^{-2}[|\mathbf{r}'_1 - \mathbf{r}'_2|^2 + |\mathbf{r}_1 - \mathbf{r}_2|^2 + |\mathbf{r}'_1 - \mathbf{r}'_2||\mathbf{r}_1 - \mathbf{r}_2|]$$

where w_0 is effective beam radius of transmitting aperture plane.

Substituting Eq. (3) into Eq. (4) yields

$$W(r'_1,\phi'_1,r'_2,\phi'_2,z) = E_0^2\left(\frac{k}{2\pi z}\right)^2 \exp\left(-\frac{\mathrm{i}k(r_1'^2 - r_2'^2)}{2z}\right) \times \int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} \exp\left(-\frac{(r_1^2+r_2^2)}{w_0^2}\right) \times \exp\left(-\frac{\mathrm{i}k}{2z}(1 - z/f)[(r_1^2 - r_2^2)]\right) \times \exp\left(\frac{\mathrm{i}kr'_1r_1}{z}\cos(\phi'_1 - \phi_1)\right)$$