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A skew ray tracing approach for the error analysis of optical elements with paraboloidal boundary surfaces

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Abstract

Using the error analysis methodology developed by the current author in previous studies for optical systems comprising elements with flat boundary surfaces, this study examines the errors induced in a light ray's path as it is reflected or refracted at a paraboloidal boundary surface. In analyzing the light path, two principal sources of error are considered, namely (1) translational errors (Δx_i, Δy_i and Δz_i) and rotational errors ($\Delta \Gamma_i$, $\Delta \Psi_i$ and $\Delta \Phi_i$), which collectively determine the deviation of the light path at each boundary surface, and (2) the differential changes induced in the incident point position and unit directional vector of the refracted/reflected ray as a result of differential changes in the position and unit directional vector of the light source. The validity of the proposed approach is verified using a generic parabaloidal boundary surface for illustration purposes. Overall, the results show that the proposed error analysis methodology provides a straightforward means of analyzing the performance of optical systems characterized by paraboloidal boundary surfaces such as headlight reflectors, optical telescope mirrors, flashlights and so forth.

Keywords: Skew ray tracing; Error analysis; Paraboloidal boundary surface

1. Introduction

Evaluating the performance of an optical system during its design stage requires the ability to determine the paths of the light rays as they undergo successive reflection and refraction events at the boundaries of the individual optical elements within the system. In general, the light path is most easily determined using some form of ray tracing technique, in which Snell's optical laws are systematically applied at each boundary surface [1,2]. Three basic types of light ray exist within any optical system, namely axial, meridional and skew. Of these three types of ray, axial and meridional rays are easily traced using simple trigonometric formulae, or even graphically if a crude approximation is sufficient. Furthermore, the paths followed by meridional rays in the paraxial regions of an optical system can be approximated via successive applications of a matrix production operation [3]. However, compared to axial rays and meridional rays, the tracing of skew rays is far more challenging. Nonetheless, skew rays are the most common form of ray in an optical system, and thus it is essential to trace their paths if the performance of the optical system is to be evaluated with any degree of reliability. Traditional skew ray tracing techniques, even with digital computers, are computationally expensive. The current group resolved this problem by developing a ray path analysis method based upon successive

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applications of a homogeneous transformation matrix and Snell's optical laws of reflection and refraction. The validity of the proposed approach was demonstrated for both flat [4] and spherical [5] optical components. In practice, however, many optical systems comprise aspherical lenses since they tend to be both smaller and lighter than their spherical counterparts and can enable the total number of optical elements within the system to be reduced. A lens in which the curved surface (or surfaces) is described by a parabola rotated in space is known as a parabolic lens. Such lenses offer a number of advantages compared with spherical lenses, including correcting the spherical aberration [6]. As a result, parabolic lenses are used in a wide variety of applications, including flashlights, automobile headlight reflectors, radio telescope antennae, microwave horns, acoustical dishes and optical telescope mirrors.

In general, ray tracing enables the sensitivity of an optical system to design or manufacturing flaws to be assessed by correlating the differential changes in the reflected or refracted rays at each boundary surface with the differential changes in the position and unit directional vector of the light source. In this way, the effect on the light path of each boundary surface within the optical system can be systematically examined. Traditionally, determining optical aberrations mathematically has always been a formidable task, involving a virtually infinite amount of ray tracing. However, sensitivity analysis enables the construction of basic aberration functions that greatly simplify the optical design task [4,5]. Moreover, the sensitivity analysis provides a convenient means of calculating the path of the chief ray; an important characteristic ray in optical systems. For example, Lin and Liao [7] performed a sensitivity analysis based on a Jacobian matrix and the Newton–Raphson method to determine the path followed by the chief ray in a binocular stereo vision system. Sensitivity analysis also enables the orientation of an image to be accurately determined. For example, Tsai and Lin [8] employed a merit function derived from a sensitivity analysis to track the change in orientation of an image as it was successively reflected/refracted at the flat boundary surfaces within a prism.

In many ways, designing an optical system is analogous to the process of designing a machine tool. In the same way that optical systems comprise a series of optical elements, machine tools comprise a series of mechanical links and joints. When designing mechanical systems, it is essential to identify the potential sources of error within the system, and to clarify their individual and combined effects, such that the quality of the machined products can be reliably determined. In 1972, Tlusty [9] examined the correlation between the overall error of a six-axis NC machine tool and the errors in each of its degrees of freedom. Ferreira and Liu [10] developed a generic model based on three rotational errors, i.e. $\Delta\Gamma_i$, $\Delta\Psi_i$ and $\Delta\Phi_i$, and three translational errors, i.e. ΔX_i, ΔY_i and ΔZ_i, to enable the systematic analysis of the machining performance of a multi-link machine tool. Taking the case of a jig-boring machine for illustration purposes, Schultshik [11] applied the same errors to investigate the combined effect of the various components of the volumetric accuracy of a machine tool on its machining precision. In a similar study, Dobowsky et al. [12] employed a parameter identification approach to identify the principal kinematic errors in planar mechanisms.

The geometrical precision of the components produced by a machine tool is dependent upon the quality and configuration of each of its links and joints. In the same way, the quality of the image produced by an optical system is determined by the errors that inevitably exist in the fabrication and assembly of the individual optical components within it. Provided that the resolution of each optical component is known, the sensitivity analysis provides the means to establish the respective contribution of each optical boundary surface along the ray path to the overall resolution of the optical system. However, ray tracing though aspherical surfaces such as those considered in the present study is difficult since the point at which the ray intersects the aspherical surface cannot be directly determined. Hence, conventional ray tracing methods do not easily lend themselves to the sensitivity analysis of optical systems comprising aspherical elements. Smith [13] attempted to resolve this problem by developing an iterative method for aspherical-boundary skew ray tracing based on a series of approximations which continued until the approximation error reduced to a negligible value. However, the method was time consuming and computationally expensive.

In recent studies, the current author developed a computationally straightforward error analysis technique based on a skew ray tracing approach for the analysis of light path errors as the light ray was reflected and/or refracted at a succession of optical elements with a flat boundary surface [14]. In the current study, this technique is extended to the error analysis of optical systems comprising elements with paraboloidal boundary surfaces. The error analysis model proposed in this study considers six specific sources of light path error, i.e. three rotational errors and three translational errors, and incorporates an error matrix that takes account not only of the effects of these errors on the deviation of the light path at each boundary surface, but also the differential changes induced in the incident point position and unit directional vector of the refracted/reflected ray as a result of differential changes in the position and unit directional vector of the light source.

In general, the homogeneous coordinate representation of a vector in 3-space is a 4-space entity specified such that a particular perspective projection recreates