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Fresnel diffraction of truncated Gaussian beam

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Abstract

We study the Fresnel diffraction of Gaussian beam truncated by one circular aperture, and give the general analytic expression of the Fresnel diffraction of truncated Gaussian beam denoted by Bessel functions. Then the characteristic of the axial diffraction fluctuation and the influence of the caliber of the circular aperture and the wave waist of Gaussian beam on the diffraction distributions are discussed, respectively. Through the numerical calculations, the characteristics of the transverse diffraction are presented and the relationship of the fluctuation of the transverse diffraction profile and the position of the axial point is shown. The physical origin of the fluctuation of Fresnel diffraction intensities of truncated Gaussian beam is expressed in terms of Fresnel half-zone theory. These phenomena and the conclusions are important for the measurement of the parameters of the beam and its applications.

Keywords: Gaussian beam; Fresnel diffraction; Fresnel half-zone theory

1. Introduction

The propagation, the diffraction and the focus of the laser beam is an important subject in the optical communication and the optical information processing, and it has been encountered in many fields. The Gaussian beam is a fundamental model of the laser beam used in practice. As it is well known, when Gaussian beam is limited by a hard-edged aperture during the propagation, which is ineluctable in practical optical system, its diffraction characteristic will be different from that of the free Gaussian beam. Therefore, the diffraction of truncated Gaussian beam has been attracted widely attentions [1–8]. Usually, we are more interested in the Fresnel diffraction of truncated Gaussian beam. Since the analytic solution can make the extreme values and the profile of the whole field clearer, it is a significant work to obtain the analytic expression of Fresnel diffraction of the truncated Gaussian beam in the study of the beam propagation. Early in 1971, Schell et al. [1], studied the diffraction of the truncated Gaussian beam in terms of Kummer's function by using a relation from Abramowitz and Stegun. After that, many discussions have studied the diffraction of truncated Gaussian beam from different aspects. Overfelt et al. [2], discussed the diffraction of Gaussian beam, Bessel beam and Bessel–Gaussian beam limited by a circular aperture recurring to the characteristic of Bessel function, and the diffraction field denoted by hypergeometric functions was given. Nourrit et al. [3], analyzed the diffraction of Gaussian beam limited by a rectangle aperture according to error functions. Toker et al. [4], expanded the diffraction field and Bessel function into Chebyshev polynomial, and Ding et al. [5], took the circular aperture function as the superposition of

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Gaussian functions. These works rich greatly the study of the diffraction of the truncated Gaussian beam and its applications.

However, in these studies, the expressions of Fresnel diffraction of truncated Gaussian beam are both given by more complex forms, and the characteristic of the axial diffraction is perspicuous but the transversal diffraction is not clear. Moreover, the physical origin of the characteristic of diffraction distribution of truncated Gaussian beam is not given out. In our former study [6], we analyze the Fresnel diffraction of the circular aperture illuminated by the parallel light. The aim of this paper is to present the general analytic formula of Fresnel diffraction of Gaussian beam limited by a circular aperture available for both the axial diffraction and the transversal diffraction, and thus the characteristics of the diffraction distribution can be discussed conveniently. According to the Fresnel zone theory, we give the respective explanation about these diffraction rules and the phenomena.

2. The diffraction theory of truncated Gaussian beam

Supposing the spatial distribution of Gaussian beam profile in the fundamental mode be expressed by $a(r_0) = L\exp(-(r_0^2/\omega_0^2))$, where L is the constant and ω_0 is wave-waist or the spot size of the Gaussian incident beam which is directly applicable to collimated laser beams, the amplitude of the field immediately after the aperture with the aperture function $p(r_0)$ can be written as

$$b(r_0) = a(r_0)p(r_0). \quad (1)$$

In the Fresnel approximation [7], the diffraction field in the observation plane r at the distance z from the aperture plane r_0, as shown in Fig. 1, is expressed by

$$A(r,z) = \frac{\exp(\mathrm{i}kz)\exp(\mathrm{i}kr^2/2z)}{\mathrm{i}z\lambda} \times \int_0^{2\pi}\int_0^{\infty} b(r_0)\exp\left(\frac{\mathrm{i}kr_0^2}{2z}\right) \times \exp\left[-\mathrm{i}\,\frac{krr_0}{z}\cos(\theta-\varphi)\right] r_0\,\mathrm{d}r\,\mathrm{d}\theta \quad (2)$$

In term of Bessel functions, the above formula can be written as

$$A(r,z) = \frac{2\pi\exp(\mathrm{i}kz)\exp(\mathrm{i}kr^2/2z)}{\mathrm{i}z\lambda} \times \int_0^{\infty} b(r_0)\exp\left(\frac{\mathrm{i}kr_0^2}{2z}\right) J_0\left(\frac{krr_0}{z}\right) r_0\,\mathrm{d}r_0. \quad (3)$$

If the second phase factor $\exp(\mathrm{i}(\pi r_0^2/\lambda z))$ in Eq. (3) can be neglected, the diffraction is often considered as the Fraunhoffer diffraction where the far divergence is

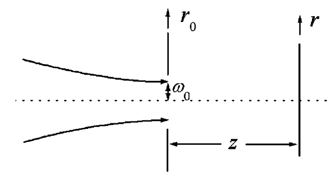

Fig. 1. The diagram of the diffraction of truncated Gaussian beam.

discussed [8]. When the aperture is a circular one with radius R, then $p(r_0) = \mathrm{circ}(r_0/\mathrm{R})$. We let Eq. (3) perform an integral operation by using of the characteristic of Bessel function and the method of the partial integral [9], then

$$A(r,z) = A_0\left\{\exp\left[\mathrm{i}\,\frac{\pi}{\lambda z}r^2\left[1-\left(1-\frac{\lambda z}{\mathrm{i}\pi\omega_0^2}\right)^{-1}\right]\right] - \exp\left[\frac{\mathrm{i}\pi(r^2+R^2)}{\lambda z}-\frac{R^2}{\omega_0^2}\right] \times \sum_{n=0}^{\infty}\left[-\mathrm{i}\,\frac{r}{R}\left(1-\frac{\lambda z}{\mathrm{i}\pi\omega_0^2}\right)^{-1}\right]^n J_n[2\pi Rr/(\lambda z)]\right\}, \quad (4)$$

where $A_0 = L\exp(\mathrm{i}kz)\left(1-(\lambda z/\mathrm{i}\pi\omega_0^2)\right)^{-1}$. This is the general formula of Fresnel diffraction of Gaussian beam truncated by a circular aperture. When $\omega_0 \to \infty$, the above equation is just the description of Fresnel diffraction of the circular aperture illuminated by the parallel light

$$A(r,z) = L\exp(\mathrm{i}kz)\left\{1-\exp\left[i\,\frac{\pi}{\lambda z}(r^2+R^2)\right] \times \sum_{n=0}^{\infty}\left[-i\,\frac{r}{R}\right]^n J_n[2\pi Rr/(\lambda z)]\right\}. \quad (5)$$

This formula is the same as the result in our former study [6]. In the following content, we will discuss, respectively, the axial diffraction and the transversal diffraction of Gaussian beam truncated by a circular aperture in Fresnel diffraction region.

3. The axial diffraction of truncated Gaussian beam

3.1. The axial diffraction of truncated Gaussian beam

When $r = 0$, and considering $J_0(0) = 1, J_n(0) = 0\ (n \neq 0)$, we can easily obtain the expression of the axial diffraction of truncated Gaussian beam from Eq. (4),

$$A(0,z) = A_0\left[1-\exp\left(\mathrm{i}\,\frac{\pi R^2}{\lambda z}-\frac{R^2}{\omega_0^2}\right)\right]. \quad (6)$$