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Numerical simulation of Bessel beams by FDTD employing the superposition principle

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Abstract

Finite difference time domain (FDTD) method is adopted to build a Bessel beams simulation model according to homogeneousness and linearity of the Maxwell equations in source-free region. Validation for this model is confirmed by comparing the simulation results with the theoretical results solved with a vector Helmholtz equation in free space and good agreement with maximum error 2% has been demonstrated. It is indicated that FDTD could be an effective approach to analyze other complicated models of Bessel beams in source-free region by means of superposition principle.

Keywords: Bessel beams; Finite difference time domain; Near-field optics

1. Introduction

Bessel beams, as one of cylindrically symmetric waves, is a diffractionless electromagnetic wave, the scalar theory of which was proposed for the first time by Durnin [1] and Durnin et al. [2]. It plays an important role in many optical aspects [3–6] as it provides convenient techniques for avoiding or reducing an inevitable diffractive spatial spreading. Vector solution of Bessel beams was further calculated by solving a vector potential and vector Helmlotz equation [7,8]. Since the nondiffracting property of Bessel beams, several research works focus on its application on near-field optics [9–11].

Since, however, most numerical algorithms in far-field optics cannot be applied in near-field optics, finite difference time domain (FDTD) method is one of the most popular algorithms adapted in near-field optics as it is well-known for solving the electromagnetic problems with arbitrary boundary conditions and inhomogeneous materials [12]. The continuous space is discretized into small cubes called Yee cells with the size less than size of relevant features and the time is discretized into small steps much less than the period of the interested electromagnetic wave so that two curl equations of Maxwell equations become difference equations. Generally, the result of this algorithm would be stable if $\Delta t \leqslant \Delta u/(\sqrt{3}v)$ according to numerical stability condition in 3D FDTD [12], where Δu is the side length of a cubic cell, Δt is the time step, v is the optical velocity in the considered medium. Mur [13] and Liao et al. [7] absorbing boundary conditions are used practically in the algorithm in order to make actual computation possible.

In this paper, exact vector solution of zero-order transversal magnetic (TM) mode Bessel beams has been

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concluded by solving vector Helmlotz equation. And then, a zero-order TM mode Bessel beams simulation model is built by means of superposition of plane wave with cylindrical linear polarization. At last, comparison of the simulation result and the theoretical solution is carried out as to the transversal and longitudinal intensity distribution at any cross-section.

2. Exact solution of Bessel beams

The vector monochromatic wave equation can be represented as the following vector Helmholz equation:

$$\nabla^2 \mathbf{C} + k^2 \mathbf{C} = 0, \tag{1}$$

where k is wave vector of a monochromatic wave and $\mathbf{C}$ could be any vector, such as electrical field, magnetic field and vector potential. According to electromagnetic theory and Bouchal and Olivik's work [5], a vector potential could be defined as the follows:

$$\mathbf{A} = \frac{\mathrm{i}}{\omega} \sum_n (a_n \mathbf{M}_n + b_n \mathbf{N}_n + c_n \mathbf{L}_n), \tag{2}$$

where $\mathbf{M}_n$, $\mathbf{N}_n$ and $\mathbf{L}_n$ are derived from the solution of scalar Helmholz equation ψ_n [8]. So magnetic induction and electrical vector could be written in this form:

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mathrm{i}}{\omega} \sum_n (b_n \mathbf{M}_n + a_n \mathbf{N}_n)$$
$$\mathbf{E} = -\sum_n (a_n \mathbf{M}_n + b_n \mathbf{N}_n). \tag{3}$$

A, **B**, **E** will satisfy vector Helmholz equation, respectively, if ψ_n is solutions of scalar Helmhotz equation so that a generalized vector solution of Bessel beams could be solved easily [5]. Then we could derive zero-order TM mode Bessel beams (shorten as Bessel beams later), or namely zero-order radially polarized wave, in the free space from the generalized solution (as shown in Fig. 1):

$$E_\rho = \mathrm{i}2\alpha J_1(\alpha\rho)\exp(\mathrm{i}\beta z),$$
$$E_\varphi = 0,$$
$$E_z = -\alpha^2 J_1(\alpha\rho)\exp(\mathrm{i}\beta z)/\beta, \tag{4}$$

where ρ, z are two cylindrical coordinates, $\alpha^2 + \beta^2 = k^2$ and k is wave vector of a monochromatic wave. Furthermore, magnetic induction of Bessel beams can be written as

$$B_\rho = B_z = 0,$$
$$B_\varphi = \mathrm{i}2J_1(\alpha\rho)\exp(\mathrm{i}\beta z)/v, \tag{5}$$

where v is the optical velocity in the considered medium. Hence, it is clearly shown that the transversal distribution of Bessel beams keeps unchanged along the z direction, which is nondiffracting property of Bessel beams in free space. And average energy flow $\mathbf{S}_{\text{average}}$ of

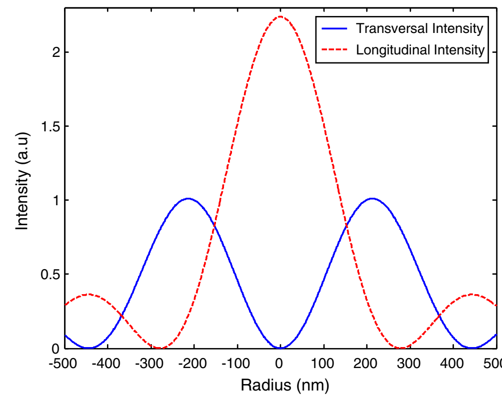

Fig. 1. Theoretical result of intensity distribution of Bessel beams in transversal and longitudinal component ($\alpha = k \sin 60^\circ$).

Bessel beams is only along the z direction, which indicates that energy propagating direction of Bessel beams is z direction and is perpendicular to its equiphase surface.

3. Numerical model of Bessel beams in FDTD

Using a commercial FDTD package (Remcom XFDTD 6.1), either plane wave or electric dipole can only be set as an excitation source. Hence, this commercial package cannot be used directly to simulate Bessel beams. However, we can build a numerical model to achieve this goal indirectly in XFDTD based on the theoretical analysis in the last section.

According to the Poisson integral formula of Bessel function, Eq. (4) can be written as

$$E_\rho = \frac{\alpha \exp(\mathrm{i}\beta z)}{2\pi} \int_0^{2\pi} \mathrm{e}^{\mathrm{i}\alpha\rho\cos\theta} \cos\theta \,\mathrm{d}\theta,$$
$$E_z = -\frac{\alpha^2 \exp(\mathrm{i}\beta z)}{\beta 2\pi} \int_0^{2\pi} \mathrm{e}^{\mathrm{i}\alpha\rho\cos\theta} \,\mathrm{d}\theta. \tag{6}$$

By defining $\alpha = k \sin\varphi$ and $\beta = k\cos\varphi$, Eq. (5) can be expressed as plane waves with cylindrical vector symmetry are interfered to generate a Bessel beams (as shown in Fig. 2). So incident plane waves with cylindrical vector symmetry are set with an identical source-free FDTD model, which is used to calculate time and again. Finally, we develop a Matlab program to superpose the calculated results of complex amplitude to form the complex amplitude of Bessel beams after compensating phase difference among plane waves as post-processing of XFDTD calculation.

Validation for this model based on XFDTD program could be confirmed by comparing the simulation result with theoretical one in the free space as analyzed previously. The incident optical waves are linear