Available online at www.sciencedirect.com
ScienceDirect
Optik 118 (2007) 330–334
Optik Optics
www.elsevier.de/ijleo

Near-field hexagonal array illumination based on fractional Talbot effect

Weijuan Qu*, Liren Liu, De'an Liu, Ya'nan Zhi, Wei Lu

Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, P.O. Box 800-211, Shanghai 201800, PR China

Received 24 January 2006; accepted 3 April 2006

Abstract

Hexagonal array is a basic structure widely exists in nature and adopted by optoelectronic device. A phase plate based on the fractional Talbot effect that converts a single expanded laser beam into a regular hexagonal array of uniformly illuminated apertures with virtually 100% efficiency is presented. The uniform hexagonal array illumination with a fill factor of 1/12 is demonstrated by the computer simulation.

Keywords: Hexagonal array illumination; Fractional Talbot effect; Fill factor; Phase plate

1. Introduction

The hexagonal array encircles a maximum area with the shortest boundary length of any equilateral polygon array. It is a nature-preferred economical pattern, such as a bee's honeycomb, carbon dioxide. And it has been widely used in optical devices such as fiber couplers [1], gradient-index rods [2], photonic delay lines [3], and cellular logic image processors [4] in optical computing. It is a nonorthogonal periodic array that cannot be represented by orthogonal arrays. Recently, Peng Xi et al. [5] have studied a hexagonal array illumination based on a phase gratings with 0 and π phase difference. It is the unique property of the hexagonal array to give an array illumination in this way. Also the hexagonal array is a periodic array, so we can get the array illumination by the fractional Talbot effects as other periodic arrays [6–8].

The Talbot array illuminators (TAIs) are periodic phase diffractive elements, which can be designed and manufactured based on the theory of the fractional Talbot effect. Because it can effectively realize the high speed and concurrent optical operation, TAIs have broad applications in areas that include optical interconnection, optical computing and optoelectronic processing. Lohmann [6] and Thomas [7] were among the first to describe array illumination based on the fractional Talbot effect with experimental demonstrations at $(1/4)Z_T$ and $(1/6)Z_T$, where $Z_T = 2T^2/\lambda$ is the Talbot distance. After that, many researchers presented equations which can be used to calculate the diffraction field at the fractional Talbot distance, such as Leger and Swanson [8], Liu [9], Arrizon and Ojeda-Castaneda [10,11], Zhou et al. [12,13]. But all those equations are used to calculate the one-dimensional grating or the two-dimensional (2D) array obtained by two orthogonal gratings. We are interested in the study of the fractional Talbot effect of the array that is obtained by two nonorthogonal gratings, which is basic to the new type array illuminator such as hexagonal array illuminator.

In Section 2, we present a mathematical description of the hexagonal array and prove the self-image of it. In Section 3, we analyze the fractional Talbot effect of the

*Corresponding author.
E-mail address: ququ@mail.siom.ac.cn (W. Qu).

0030-4026/$ - see front matter © 2006 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2006.04.006

hexagonal array with different fill factor in detail. Finally, we give a calculated phase distribution in the fractional Talbot plane of the hexagonal array with fill factor of 1/12, with which a hexagonal array illumination can be obtained.

2. The Talbot effect of 2D amplitude-type hexagonal array

As shown by Fig. 1, in the amplitude-type hexagonal array a black hexagon corresponds to luminescence and a white hexagon corresponds to nonluminescence. Assuming that the amplitude transmittance of the hexagonal array is

$$t(x,y) = t_c(x,y) * \varphi(x,y), \tag{1}$$

where

$$t_c(x,y) = \mathrm{rect}\left(\frac{\sqrt{3}x+y}{d}\right)\mathrm{rect}\left(\frac{\sqrt{3}x-y}{d}\right)\mathrm{rect}\left(\frac{2y}{\sqrt{3}d}\right)$$

is the amplitude transmittance of the hexagonal cell, the length of the hexagonal lateral is $d/2$,

$$\varphi(x,y) = \mathrm{comb}\left(\frac{\sqrt{3}x+y}{T}, \frac{\sqrt{3}x-y}{T}\right)$$

is the lattice-generating function, which can generate an spot array of period T in the two directions that parallel to the line $\sqrt{3}x+y=0$ and $\sqrt{3}x-y=0$, $*$ denotes the convolution operation.

The amplitude-type hexagonal array is normally illuminated by a unit-amplitude plane wave. We use the Fresnel transform formula to evaluate the amplitude distribution of the observation plane at distance z. The Fresnel transform of a two-dimensional (2D) function is defined by a convolution with a scaled quadratic phase function

$$f(x,y,z) = \frac{\exp(jkz)}{j\lambda z} t_c(x,y) * \Delta(x,y), \tag{2}$$

where $\Delta(x,y) = \varphi(x,y) * h(x,y)$, $h(x,y) = \exp[j\pi(x^2 + y^2)/\lambda z]$ is the optical transfer function in the Fresnel domain. We can write the comb function in terms of the Fourier series of delta function, so $\varphi(x,y)$ is

$$\begin{aligned}\varphi(x,y) &= \mathrm{comb}\left(\frac{\sqrt{3}x+y}{T}, \frac{\sqrt{3}x-y}{T}\right)\\ &= |T|^2 \sum_n \sum_m \delta(\sqrt{3}x+y-nT)\delta(\sqrt{3}x-y-mT)\\ &= \frac{1}{|T|^2}\sum_n \sum_m \exp\left[i2\pi n\left(\frac{\sqrt{3}x+y}{T}\right)\right]\\ &\quad \times \exp\left[i2\pi m\left(\frac{\sqrt{3}x-y}{T}\right)\right].\end{aligned}$$

According to convolution theorem and the property of the delta function, we can compute $\Delta(x,y)$ as follows:

$$\begin{aligned}\Delta(x,y) =& F^{-1}\left\{\sum_n \sum_m \delta\left(f_x - \frac{\sqrt{3}n+\sqrt{3}m}{T}\right)\delta\left(f_y - \frac{n-m}{T}\right)\right.\\ &\left.\times \exp\left[-j\pi\lambda z\left(\left(\frac{\sqrt{3}n+\sqrt{3}m}{T}\right)^2 + \left(\frac{n-m}{T}\right)^2\right)\right]\right\}.\end{aligned} \tag{3}$$

We define a new function as

$$C(n,m) = \exp\left[-j2\pi\frac{z}{T^2/2\lambda}(n^2+m^2-mn)\right] \quad \text{and} \quad n^2+m^2-mn$$

is integer. The inverse Fourier transform in Eq. (3) can be evaluated using the Fourier series expression for delta function. Hence $\Delta(x,y)$ becomes

$$\begin{aligned}\Delta(x,y) =& \frac{T^2}{|2\sqrt{3}|}\sum_n \sum_m C(n,m)\\ &\times \iint_\infty e^{j\pi n(-f_y-(f_x)/\sqrt{3})T}\, e^{j\pi m(f_y-(f_x)/\sqrt{3})T}\\ &\times e^{j2\pi(f_x x+f_y y)}\,df_x\,df_y.\end{aligned} \tag{4}$$

Evaluate the integration of Eq. (4), finally we can obtain the expression of $\Delta(x,y)$ as follows:

$$\begin{aligned}\Delta(x,y) =& |T|^2 \sum_n \sum_m C(n,m)\delta(\sqrt{3}x+y-nT)\delta\\ &\times(\sqrt{3}x-y-mT),\end{aligned} \tag{5}$$

Obviously, if $z = lT^2/2\lambda$, l is integer, we can obtain the following terms:

$$\begin{aligned}\Delta(x,y) =& |T|^2 \sum_n \sum_m \delta(\sqrt{3}x+y-nT)\\ &\times \delta(\sqrt{3}x-y-mT).\end{aligned} \tag{6}$$

It is a spot array same as the spot distribution described by function $\varphi(x,y)$. Therefore, the Talbot distance of the amplitude-type hexagonal array is $z_T = T^2/2\lambda$, where T is the above period of the lattice, λ is the wavelength of the illuminated light.

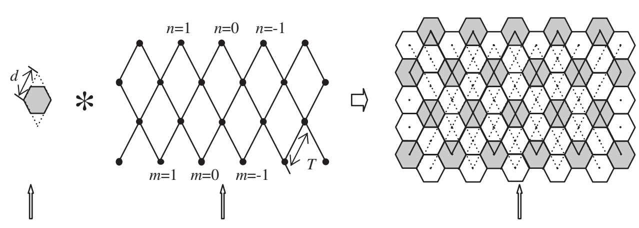

Fig. 1. The generation of the 2D hexagonal array, $*$ denotes the convolution operation.