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Spectral anomalies of diffracted chirped Gaussian pulses and their potential applications

Qihui Zou[a,b], Baida Lü[a,*]

[a]Institute of Laser Physics & Chemistry, Sichuan University, Chengdu 610064, China
[b]College of sciences of Southwest Petroleum University, Chengdu 610500, China

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Abstract

Starting from the Rayleigh–Sommerfeld diffraction integral and without invoking the paraxial approximation, analytical expressions for the field distribution, far-field power spectrum and temporal far-field distribution of chirped Gaussian pulses diffracted at a circular aperture are derived, which enables us to study the spectral anomalous behavior of diffracted chirped Gaussian pulses in the far field. The potential applications of spectral anomalies of ultrashort pulses are discussed. It is found that at the critical angle the spectral switch appears. The frequency difference between the two equal heights of spectral switches increases and the corresponding critical diffraction angle slightly increases as the chirp parameter increases and pulse duration decreases. In a certain region of the truncation parameter, the critical angle decreases with increasing truncation parameter. By suitably varying the pulse duration, chirp parameter and truncation parameter, information encoding and transmission are achievable in the use of chirped Gaussian pulses.

Keywords: Singular optics; Spectral anomaly; Chirped Gaussian pulse; Diffraction at a circular aperture

1. Introduction

There has been much interest in singular optics, which deals with a variety of effects in spatially coherent monochromatic wavefields at points of zero intensity, where the phase becomes indeterminate [1]. Recently, the subject of singular optics has been extended to spatially coherent, polychromatic wavefields and partially coherent quasi-monochromatic wavefields [2]. Specifically, the spectral anomalies near phase singularities in polychromatic beams have been extensively studied both theoretically and experimentally [3–8]. However, most of the works in the spectral anomalies have been restricted to the polychromatic steady-state beams within the framework of the paraxial approximation. The aim of the present paper is to study spectral anomalies near phase singularities of chirped Gaussian pulses diffracted at a circular aperture and their potential applications. In Section 2, starting from the Rayleigh–Sommerfeld diffraction integral, the closed-form expressions for the field distribution, far-field power spectrum and temporal far-field distribution of ultrashort chirped Gaussian pulses diffracted at a circular aperture are derived and analyzed. Section 3 presents numerical calculation results and analyses of spectral anomalies of diffracted chirped Gaussian pulses in the far field. The potential applications of spectral

*Corresponding author. Fax: +86 28 8540 3260.
E-mail address: baidalu0@tom.com (B. Lü).

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anomalies of ultrashort pulses in the information encoding and transmission are discussed in Section 4. Finally, Section 5 summarizes the main results obtained in this paper.

2. Basic equations

Assume that in the cylindrical coordinate system at the plane $z = 0$ there is a Gaussian pulse of the form

$$E_0(\rho_0, 0, \omega) = S(\omega)\exp(-\rho_0^2/w_0^2), \tag{1}$$

where $S(\omega)$ denotes the original spectrum on the axis and w_0 is the waist width that is assumed to be independent of the frequency ω [9]. The Gaussian pulse passes through a circular aperture of the radius a at the plane $z = 0$. According to the Rayleigh–Sommerfeld diffraction integral, the field in the half-space $z > 0$ reads as

$$\begin{aligned} E(r, \theta, \omega) = {} & \frac{\mathrm{i}k \cos\theta}{2\pi} \frac{\mathrm{e}^{-\mathrm{i}kr}}{r} \int_0^a \int_0^{2\pi} E_0(\rho_0, 0, \omega) \\ & \times \exp\left(-\frac{\mathrm{i}k}{2r}\rho_0^2\right) \\ & \times \exp\left[\frac{\mathrm{i}k}{r}\rho\rho_0 \cos(\varphi - \varphi_0)\right]\rho_0\,\mathrm{d}\rho_0\,\mathrm{d}\varphi_0, \end{aligned} \tag{2}$$

where $\cos\theta = z/r$, $r = (\rho^2 + z^2)^{1/2}$, θ is the diffraction angle and k is the wave number related to the frequency ω and speed of light in vacuum c by $k = \omega/c$. It is noted that the paraxial approximation is not used in Eq. (2). As a result, Eq. (2) is applicable to the more general case.

The substitution from Eq. (1) into Eq. (2) yields

$$\begin{aligned} E(r, \theta, \omega) = {} & \mathrm{i}k \cos\theta \frac{\mathrm{e}^{-\mathrm{i}kr}}{r} S(\omega) \int_0^a \exp\left[-\left(\frac{1}{w_0^2} + \frac{\mathrm{i}k}{2r}\right)\rho_0^2\right] \\ & \times J_0(k\rho_0 \sin\theta)\rho_0\,\mathrm{d}\rho_0, \end{aligned} \tag{3}$$

with $J_0(\cdot)$ being the zeroth-order Bessel function.

The integration of Eq. (3) leads to

$$\begin{aligned} E(r, \theta, \omega) = {} & \mathrm{i}kS(\omega)\cos\theta \frac{\mathrm{e}^{-\mathrm{i}kr}}{r} \sum_{s=0}^{\infty} (-1)^s \frac{\omega^{2s} w_0^{2s+2} \sin^{2s}\theta}{2^{2s+1}(s!)^2 c^{2s}} \\ & \times \delta^{2s+2}\left[\delta^2\left(1 + \frac{\mathrm{i}z_0}{r}\right)\right]^{-(s+1)} \\ & \times \left\lfloor s! - \Gamma(s+1, \delta^2(1 + \mathrm{i}z_0/r))\right\rfloor, \end{aligned} \tag{4}$$

where $\Gamma(\bullet,\bullet)$ denotes the incomplete gamma function, $\delta = a/w_0$ is the truncation parameter and $z_0 = kw_0^2/2$ is the Rayleigh length. Eq. (4) is the analytical propagation equation of ultrashort Gaussian pulses in the frequency domain, which is applicable to both Fresnel and Fraunhofer regions.

In the far-field approximation, Eq. (4) simplifies to

$$\begin{aligned} E(r, \theta, \omega) = {} & \mathrm{i}kS(\omega)\cos\theta \frac{\mathrm{e}^{-\mathrm{i}kr}}{r} \sum_{s=0}^{\infty} (-1)^s \\ & \times \frac{\omega^{2s} w_0^{2s+2} \sin^{2s}\theta}{2^{2s+1}(s!)^2 c^{2s}} [s! - \Gamma(s+1, \delta^2)]. \end{aligned} \tag{5}$$

The power spectrum in the far field turns out to be

$$|E(r, \theta, \omega)|^2 = |S(\omega)|^2 M(r, \theta, \omega), \tag{6}$$

where

$$\begin{aligned} & M(r, \theta, \omega) \\ & = \frac{\omega^2 \cos^2\theta}{r^2 c^2} \\ & \quad \times \left[\sum_{s=0}^{\infty} (-1)^s \frac{\omega^{2s} w_0^{2s+2} \sin^{2s}\theta}{2^{2s+1}(s!)^2 c^{2s}} (s! - \Gamma(s+1, \delta^2))\right]^2 \end{aligned} \tag{7}$$

is the spectral modifier describing how the aperture diffraction modifies the original spectrum $S(\omega)$.

Suppose that the incident pulse takes a chirped Gaussian form [10]

$$A(t) = \exp\left[-\frac{(1 + \mathrm{i}C)t^2}{2T^2}\right]\exp(-\mathrm{i}\omega_c t), \tag{8}$$

where T is the pulse duration defined as the half-width at $1/e$ intensity point, and is related to the full-width at half-maximum (FWHM) by $T_{\mathrm{FWHM}} = 2\,T(\ln 2)^{1/2}$, ω_c is the carrier frequency and C is the chirp parameter.

Making use of the Fourier transform of $A(t)$, we obtain the original spectrum

$$\begin{aligned} S(\omega) & = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(t)\exp(\mathrm{i}\omega t)\,\mathrm{d}t \\ & = \frac{T}{\sqrt{1 + \mathrm{i}C}} \exp\left[-\frac{T^2(\omega - \omega_c)^2}{2(1 + \mathrm{i}C)}\right]. \end{aligned} \tag{9}$$

On substituting from Eq. (9) into Eq. (6), the power spectrum of chirped Gaussian pulses in the far field reads as

$$\begin{aligned} & |E(r, \theta, \omega)|^2 \\ & = \frac{T^2}{(1 + C^2)^{1/2}} \frac{\omega^2 \cos^2\theta}{r^2 c^2} \exp\left[-\frac{T^2(\omega - \omega_c)^2}{1 + C^2}\right] \\ & \quad \times \left[\sum_{s=0}^{\infty} (-1)^s \frac{\omega^{2s} w_0^{2s+2} \sin^{2s}\theta}{2^{2s+1}(s!)^2 c^{2s}} (s! - \Gamma(s+1, \delta^2))\right]^2. \end{aligned} \tag{10}$$

Eq. (10) indicates that $|E(r, \theta, \omega)|^2$ depends on the chirp parameter C, pulse duration T, truncation parameter δ and diffraction angle θ.