Available online at www.sciencedirect.com
ScienceDirect
Optik 119 (2008) 540–544
Optik Optics
www.elsevier.de/ijleo

Phase-dependent properties for absorption and dispersion in a closed equispaced three-level ladder system

X.J. Fan[a,*], A.Y. Li[a], F.G. Bu[a], H.X. Qiao[a], J. Du[b], Z.Z. Xu[c]

[a]*College of Physics and Electronics, Shandong Normal University, Jinan 250014, China*
[b]*Communication College, Shandong Normal University, Jinan 250014, China*
[c]*Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

Received 17 August 2006; accepted 3 December 2006

Abstract

It is shown that in a closed equispaced three-level ladder system, by controlling the relative phase of two applied coherent fields, the conversion from absorption with inversion to lasing without inversion (LWI) can be realized; a large index of the refraction with zero absorption can be gotten; considerable increasing of the spectrum region and value of the LWI gain can be achieved. Our study also reveals that the incoherent pumping will produce a remarkable effect on the phase-dependent properties of the system. Modifying value of the incoherent pumping can change the property of the system from absorption to amplification and enhance significantly LWI gain. If the incoherent pumping is absent, we cannot get any gain for any value of the relative phase.

PACS: 42.50.Gy; 42.50.Hz

Keywords: Spontaneously generated coherence; Relative phase; Lasing without inversion; Absorption; Dispersion

1. Introduction

A kind of coherence can be created by interference of spontaneous emission (usually called as spontaneously generated coherence (SGC)) of either two close lying atomic levels to a common atomic level (V-type atom)[1,2] or by a single excited level to two close lying atomic levels (Λ-type atom) [3]. In a ladder type system, it can also be created in the nearly spaced atomic levels case [4]. Recently there have been considerable interests in studying the SGC [5–18]; moreover, it has been shown that atomic systems with SGC are sensitive to the relative phase of the applied fields [19–30]. Very recently, Refs. [12,15,16] have investigated effects of SGC on population inversion, optical bistability and lasing without inversion (LWI) in a closed equispaced three-level ladder system with coherent weak probe and strong driving fields, respectively. In this paper, we study effects of the relative phase between the probe and driving fields on the absorption and dispersion properties in this system and modulation role of incoherent pumping on the phase-dependent properties.

2. Model and equations

The closed equispaced three-level ladder atomic system considered here is shown in Fig. 1. The transition $|1\rangle \rightarrow |2\rangle$ is coupled by a weak probe field of frequency ω_a with *Rabi* frequency $g = \vec{\mu}_{12} \cdot \vec{\varepsilon}_a/\hbar$ while the

*Corresponding author.
E-mail address: xjfan108@yahoo.com.cn (X.J. Fan).

0030-4026/$ - see front matter © 2007 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2006.12.018

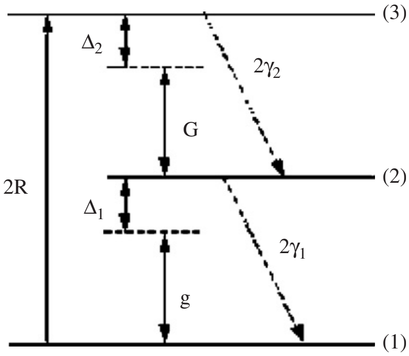

Fig. 1. A three-level ladder system with nearly equispaced levels.

transition $|2\rangle \to |3\rangle$ is coupled by a strong driving field of frequency ω_b with *Rabi* frequency $G = \vec{\mu}_{23} \cdot \vec{\varepsilon}_b/\hbar$. The level $|2\rangle(|3\rangle)$ spontaneously decays to the level $|1\rangle(|2\rangle)$ at the rate $2\gamma_1$ $(2\gamma_2)$. An incoherent pump with a pumping rate $2R$ is applied between levels $|1\rangle$ and $|3\rangle$. In such a case, the density-matrix motion equations in a rotating frame can be written as

$$\dot{\rho}_{11} = -2R\rho_{11} + 2\gamma_1\rho_{22} + \mathrm{i}g^*\rho_{21} - \mathrm{i}g\rho_{12}, \tag{1a}$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{33} - 2\gamma_1\rho_{22} - \mathrm{i}g^*\rho_{21} + \mathrm{i}g\rho_{12} - \mathrm{i}G\rho_{23} + \mathrm{i}G^*\rho_{32}, \tag{1b}$$

$$\dot{\rho}_{33} = 2R\rho_{11} - 2\gamma_2\rho_{33} + \mathrm{i}G\rho_{23} - \mathrm{i}G^*\rho_{32}, \tag{1c}$$

$$\dot{\rho}_{23} = -(\gamma_1 + \gamma_2 + \mathrm{i}\Delta_2)\rho_{23} + \mathrm{i}G^*(\rho_{33} - \rho_{22}) + \mathrm{i}g\rho_{13}, \tag{1d}$$

$$\dot{\rho}_{12} = -(R + \gamma_1 + \mathrm{i}\Delta_1)\rho_{12} + \mathrm{i}g^*(\rho_{22} - \rho_{11}) - \mathrm{i}G\rho_{13} + 2p\sqrt{\gamma_1\gamma_2}\eta\rho_{23}, \tag{1e}$$

$$\dot{\rho}_{13} = -[\gamma_2 + R + \mathrm{i}(\Delta_1 + \Delta_2)]\rho_{13} - \mathrm{i}G^*\rho_{12} + \mathrm{i}g^*\rho_{23} \tag{1f}$$

constrained by $\rho_{11}+\rho_{22}+\rho_{33}=1$ and $\rho^*_{mn} = \rho_{nm}$. Where, ρ_{ii} is the atomic population of state $|i\rangle$, and ρ_{ij} is the atomic polarization between states $|i\rangle$ and $|j\rangle$; $\Delta_1 = \omega_{21} - \omega_a$ and $\Delta_2 = \omega_{32} - \omega_b$ denote the frequency detuning of the probe and the driving fields, respectively. Here in the case of nearly equispaced levels, the inclusion of two coupling fields of different frequencies would lead to the optical Bloch equation with additional term $2p\sqrt{\gamma_1\gamma_2}\eta\rho_{23}$, which presents the effect of SGC. Where $p = \vec{\mu}_{12} \cdot \vec{\mu}_{23}/|\vec{\mu}_{12}||\vec{\mu}_{23}| = \mathrm{con}\,\theta$, θ is the angle between the two induced dipole moments $\vec{\mu}_{12}$ and $\vec{\mu}_{23}$. Using the restriction that each of the linearly polarized field should only couple one of the optical transitions, we can find that the Rabi frequencies are connected to the parameter p by the relation $G = G_0\sqrt{1-p^2} = G_0\sin\theta$ and $g = g_0\sqrt{1-p^2} = g_0\sin\theta$, G_0 and g_0 are the Rabi frequencies at the situation of no SGC. It is obvious that when $\eta = 1$, the effect of SGC presents, the strength of SGC will vary with θ; otherwise $\eta = 0$, the effect of SGC is absent. Due to SGC, the properties of the system depend not only on amplitudes and detunings but also phase of the probe and driving fields, thus we have to treat Rabi frequencies as complex parameters. Let ϕ_p and ϕ_c denote the phases of the probe and driving fields, respectively, then we have $g = g_\mathrm{p}\exp(\mathrm{i}\phi_\mathrm{p})$ and $G = G_\mathrm{c}\exp(\mathrm{i}\phi_\mathrm{c})$ (g_p and G_c are real parameters) and the relative phase between the probe and the driving fields is $\Phi = \phi_\mathrm{p} - \phi_\mathrm{c}$. Let $\tilde{\rho}_{ii} = \rho_{ii}$, $\tilde{\rho}_{12} = \rho_{12}\exp(\mathrm{i}\phi_\mathrm{p})$, $\tilde{\rho}_{23} = \rho_{23}\exp(\mathrm{i}\phi_\mathrm{c})$, $\tilde{\rho}_{13} = \rho_{13}\exp(\mathrm{i}\phi)$ and $\phi = \phi_\mathrm{c} + \phi_\mathrm{p}$, we can get equations for $\tilde{\rho}_{ij}(i,j = 1,2,3)$ which are found to be identical to Eq. (1) except that η is replaced by $\eta_\Phi = \eta\exp(\mathrm{i}\Phi)$. The steady-state solutions can be found by setting the time derivatives to zero and reducing the equations for $\tilde{\rho}_{ij}(i,j = 1,2,3)$ to a set of coupled 9×9 algebraic equations after splitting into real and imaginary parts. These equations can be treated in all orders using the symbolic computation package Mathematica or Maple. The dispersion and absorption (gain) of the medium are determined by the real and imaginary parts of $\tilde{\rho}_{12}$, respectively. If $\mathrm{Im}(\tilde{\rho}_{12}) > 0$, the system exhibits gain for the probe field; if $\mathrm{Im}(\tilde{\rho}_{12}) < 0$, the probe field is attenuated. When $\mathrm{Im}(\tilde{\rho}_{12}) > 0$ and $\tilde{\rho}_{22} - \tilde{\rho}_{11} < 0$, LWI can be realized; if $\mathrm{Im}(\tilde{\rho}_{12}) > 0$ and $\tilde{\rho}_{22} - \tilde{\rho}_{11} > 0$, the lasing with inversion occurs.

3. Numerical analysis

In the following we analyze control role of the relative phase on the properties of the system by numerical calculation result from the steady analytical solutions for $\mathrm{Im}\tilde{\rho}_{12}$, $\mathrm{Re}\tilde{\rho}_{12}$, and $\tilde{\rho}_{22} - \tilde{\rho}_{11}$.

Let us first consider the dependence of the absorption and dispersion properties of the probe field on the relative phase Φ. Fig. 2 illustrates $\mathrm{Im}\tilde{\rho}_{12}$, $\mathrm{Re}\tilde{\rho}_{12}$ and

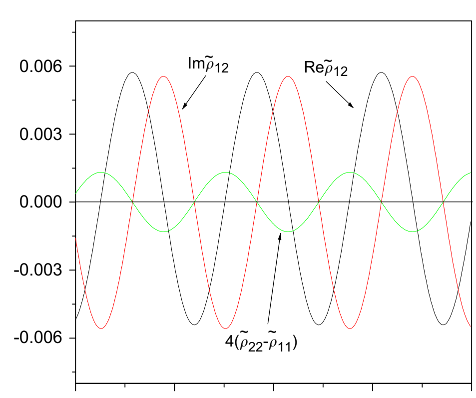

Fig. 2. $\mathrm{Im}\tilde{\rho}_{12}$, $\mathrm{Re}\tilde{\rho}_{12}$ and $\tilde{\rho}_{22} - \tilde{\rho}_{11}$ versus Φ. The parameters values are $\Delta_2 = 0$, $\Delta_1 = 5$, $\gamma_1 = 1.2\gamma_2$, $\theta = \pi/4$, $\mathrm{Gc} = 5\sin\theta\,\gamma_2$, $g_\mathrm{p} = 0.1\sin\theta\,\gamma_2$, $R = 1.2\gamma_2$ and $\eta = 1$.