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ScienceDirect
ELSEVIER
Optik 119 (2008) 127–133
Optik Optics
www.elsevier.de/ijleo

Retrieving the complex degree of spatial coherence of electron beams

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Received 12 May 2006; accepted 3 July 2006

Abstract

The possibility to characterize the coherence properties of an electron source is presented. The method, based on the determination of centered-reduced moments of the beam spot, allows the evaluation of both amplitude and phase of the complex degree of spatial coherence. The experimental results are in agreement with a different approach based on the Fourier analysis and with calculations according to the Van Cittert—Zernike theorem.

PACS: 07.60.Ly; 07.78. + s; 41.75.Fr; 42.25.Kb

1. Introduction

Partial spatial coherence of particle beams has been theoretically predicted as a result of the indistinguishability of the individual paths of neighboring particles of the beam. It means that the paths of nearby particles cannot be distinguished from each other, and therefore, their associated wave functions are correlated to some extent. This correlation, which embodies the concept of spatial coherence of the beam, is used for the interpretation of interference patterns obtained in basic and applied experiments with charged particles [1,2] neutrons and even molecules ([3] and references therein).

In view of the widespread use of electron microscopes in material science problems, the availability of electron beams with the highest possible coherence is critical for successful interference experiments. For this purpose a complex degree of spatial coherence can be defined and should be measured for properly characterizing the beam for various applications such as electron holography [1]. In general, the conventional procedures to measure the complex degree of spatial coherence are based on Young interferometry and Fourier analysis. However, these methods are not able to provide two-dimensional measurements with the required precision. Recently a different approach has been developed for the two-dimensional retrieval, both in amplitude and phase, of the complex degree of spatial coherence of a laser beam [4–6]. The method consists of the following three steps: recording the beam spot, determining its

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0030-4026/$ - see front matter © 2006 Published by Elsevier GmbH.
doi:10.1016/j.ijleo.2006.07.007

centered-reduced moments and inserting them as coefficients in a Taylor series. This procedure is simple, fast and of higher performance than conventional procedures.

In the present article we discuss the applicability of this method to the case of the electron beam provided by a field emission source of an electron microscope. At this preliminary stage, the electron optical system is considered to be aberration free in order to assess the validity of the centered-reduced moment method as well as for the sake of simplicity. This approximation is amply supported by the experimental results, which are in good agreement with those obtained with the Fourier method.

It is worthwhile emphasizing that the modulus of the complex degree of spatial coherence of an electron beam has been evaluated quantitatively from the visibility of the interference fringes obtained by performing a Young double slit experiment or using a more versatile device such as an electrostatic biprism (for a review see [7,8] and references therein).

In this way however, it is possible to obtain a measurement of the phase of the complex degree of coherence at best for a pair of points.

The centered-reduced moment method discussed in this article presents several advantages over the methods related to Young interferometry: the required electron optical configuration is much easier to set, there is no need to prepare a sample with a double slit, or to construct and utilize an electrostatic biprism. The method proposed here provides an experimental two-dimensional representation of both the modulus and the phase of the complex degree of coherence, an information that was not yet available in electron optics.

2. Fundamentals

The complex degree of spatial coherence is well defined in optics [9,10] for describing the correlation between the optical field fluctuations at two different points of a specific plane. Its modulus takes values in the interval [0,1], with zero for uncorrelated fluctuations and one if their correlation is maximal.

On the other hand, the concept of *coherence* is applied to particles, which move along indistinguishable paths and give rise to interference patterns, where indistinguishability is understood in the sense of quantum mechanics [11,12]. In this case, the wave functions of the particles will be correlated to some extent, and a complex degree of spatial coherence can be defined to describe this feature. Indistinguishability is also established when wave packets associated with two particles cannot be resolved, i.e., the wave packets are superimposed such that we cannot follow the individual particles in specific and differentiable trajectories [12]. Thus, coherence properties of a particle beam establish the fundamental link between the wave and the particle descriptions [12].

Correlation of the optical field fluctuations (in the case of light beams), or of the particle wave functions (in the case of particle beams), determines the capability of the beam to form an interference pattern. Specifically, the modulus of the complex degree of spatial coherence will be related to the visibility of the interference fringes, and its phase to shifts of the fringes with respect to the co-ordinate origin. However, it is possible to retrieve the complex degree of spatial coherence of optical fields, in both amplitude and phase, by applying a non-interferometric procedure, i.e. by using the centered-reduced moments of the spot [4–6]. In this paper, we discuss the applicability of this procedure in determining the complex degree of spatial coherence of electron beams in a field-free domain.

If the electrons of the beam are completely indistinguishable, their propagation is described by the solution of the time-independent Schrödinger equation [7]. This equation takes the form of the Helmholtz equation

$$\nabla^2 \Psi(\mathbf{r}) + k^2 \Psi(\mathbf{r}) = 0, \tag{1}$$

where $\Psi(\mathbf{r})$ is the wave function for electrons, which depend on the position vector $\mathbf{r}$, and $k = g/\hbar = 2\pi/\lambda$, g being the constant kinetic momentum and λ the corresponding De Broglie's wavelength. Following the optical equivalent [9], the solution of Eq. (1) should fulfill the Kirchhoff's general diffraction formula. By regarding the electron beam in paraxial approximation this formula takes the form used for describing Fraunhofer diffraction, i.e.

$$\Psi(x', y') = -\frac{\mathrm{i}}{\lambda z}\, \mathrm{e}^{\mathrm{i}kz} \iint_A \Psi_0(x, y)\, \mathrm{e}^{-\mathrm{i}(k/z)(x'x+y'y)}\, \mathrm{d}x\, \mathrm{d}y \tag{2}$$

with (x,y) and (x',y') the Cartesian co-ordinates on planes orthogonal to the direction of propagation z. This wave function should be of deterministic nature.

However, from a more realistic point of view, the wave function $\Psi(\mathbf{r}, t)$ should be a random variable that represents a fluctuating electron beam in space-time $(\mathbf{r},t)$. So, the behavior of the beam can be more appropriately described by the correlation of the wave function at two points of the space-time $(\mathbf{r}_1, t_1)$ and $(\mathbf{r}_2, t_2)$, i.e.

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, t_1, t_2) = \left\langle \Psi^*(\mathbf{r}_1, t_1) \Psi(\mathbf{r}_2, t_2) \right\rangle, \tag{3}$$

where the angular brackets represent ensemble average, in the sense of the second-order spatial coherence theory [7,9]. Eq. (3) indicates that the electron beam should be a stochastic process. In a field-free domain, it can be considered as stationary and ergodic, at least in the wide