Petroleum 1 (2015) 231–236

Contents lists available at ScienceDirect

Petroleum

journal homepage: www.keaipublishing.com/en/journals/petlm

Original article

Analysis for pressure transient of coalbed methane reservoir based on Laplace transform finite difference method

Lei Wang [a,*], Hongjun Yin [a], Xiaoshuang Yang [b], Chuncheng Yang [b], Jing Fu [c]

[a] Northeast Petroleum University Key Laboratory of Ministry of Education PRC, Heilongjiang, Daqing, 163318, China
[b] PetroChina Daqing Oilfield Limited Company, Heilongjiang, Daqing, 163511, China
[c] Geological Science & Engineering of Missouri University of Science and Technology, Rolla, MO, 65401-6540, USA

ARTICLE INFO

Article history:
Received 24 June 2015
Received in revised form
11 August 2015
Accepted 31 August 2015

Keywords:
Coalbed methane
Pressure performance
Fractal medium
Laplace transform finite difference method

ABSTRACT

Based on fractal geometry, fractal medium of coalbed methane mathematical model is established by Langmuir isotherm adsorption formula, Fick's diffusion law, Laplace transform formula, considering the well bore storage effect and skin effect. The Laplace transform finite difference method is used to solve the mathematical model. With Stehfest numerical inversion, the distribution of dimensionless well bore flowing pressure and its derivative was obtained in real space. According to compare with the results from the analytical method, the result from Laplace transform finite difference method turns out to be accurate. The influence factors are analyzed, including fractal dimension, fractal index, skin factor, well bore storage coefficient, energy storage ratio, interporosity flow coefficient and the adsorption factor. The calculating error of Laplace transform difference method is small. Laplace transform difference method has advantages in well-test application since any moment simulation does not rely on other moment results and space grid.

1. Introduction

Coalbed methane well test is one of the important means to obtain coalbed methane reservoir parameters, it is also an effective method to validate the flow mechanism and of coal seam directly. The medium of coal seam is typical natural fracture system which is made up of cleat and substrate. A great number of experimental data indicate that the porous medium system in coal reservoirs has the different heterogeneous structure in different scales, which show a kind of self-similarity and have the fractal characteristics [1–3]. In 1990, Chang and Yortsos [4] built up a theoretical model for fractal reservoirs and opened the door to the study of seepage mechanics in fractal reservoirs. After this, Beier et al. [5–7] used the percolation model of fractal reservoirs to explain the well test results in the complex reservoirs which couldn't be explained by traditional model. In 2007, He Yingfu, Yang Zhengming et al. [8] used the finite difference method to solve pressure transient of Coalbed methane fractal medium, but the grid effect of the method is serious and the step length of time and meshing generation is quite sensitive. The accuracy of calculation results is affected. In 2007, GAO Huimei, He Yingfu et al. [9] used analytical method to analyze pressure transient in Coalbed methane deformable medium. In 2008, Zhang Xianmin and Tong Dengke [10] also used the analytical method to study pressure dynamic in Coalbed methane fractal medium. But the analytical method has some limitations to solve directly the complex nonlinear partial differential equations. In 2012, Liu Hong, Wang Xinhai et al. [11] used the Laplace transform finite difference method to solve the one-dimensional seepage problems, and proved the applicability for calculating seepage problem.

This article established fractal medium of coalbed methane mathematical model by introducing fractal dimension and fractal index, considering the well bore storage effect and skin effect, and used Laplace transform finite difference method to solve, eliminated the time variable and weakened the effect of

* Corresponding author.
E-mail address: wangleidqpi@163.com (L. Wang).
Peer review under responsibility of Southwest Petroleum University.

Production and Hosting by Elsevier on behalf of KeAi

http://dx.doi.org/10.1016/j.petlm.2015.08.002
2405-6561/

grid. Its calculation results are more accurate than finite difference method, and it can also be applied to calculate well bore flowing pressure of the complex Coalbed methane reservoir conditions. We can analyze the influence of fractal dimension, fractal index, skin factor, well bore storage coefficient and energy storage ratio, interporosity flow coefficient and the adsorption factor on coalbed methane pressure dynamic.

2. Mathematical model

Based on the characteristics, mathematical model of Coalbed methane is established for the assumption: (1) the top and bottom boundary of the fractal reservoir are sealed, fracture network is embedded in the 2D Euclidean bedrock, the fractal dimension is d_f, (2) the flow in the cleat system is laminar flow and radial flow, and can be described by Darcy's law, (3) ignore the gravity, (4) Langmuir isothermal adsorption, (5) the fracture network still have fractal characteristics when it occurred small deformation under the pressure, and the change of the fractal dimension can be ignored.

The fractal reservoir permeability and porosity is

$$K = K_w\left(\frac{r}{r_w}\right)^{d_f-\theta-2}$$

$$\phi = \phi_w\left(\frac{r}{r_w}\right)^{d_f-2}$$

where d_f is the fractal dimension, θ is the fractal index, r_w is the radius of the bore hole, m, K_w is the reservoir permeability near the bore hole, m^2, ϕ_w is the reservoir porosity near the bore hole.

According to the motion equation, the continuity equation, and the constitutional equation of actual gas, the dimensionless governing equation of Coalbed methane in the fractal reservoir can be expressed as

$$\frac{\partial^2 m_D}{\partial r_D^2} + \frac{\beta}{r_D}\frac{\partial m_D}{\partial r_D} = r_D^{\theta}\left[\omega\frac{\partial m_D}{\partial t_D} - (1-\omega)\frac{\partial V_D}{\partial t_D}\right] \tag{1}$$

$$\frac{\partial V_D}{\partial t_D} = \frac{1}{\lambda}(V_{ED} - V_D) \tag{2}$$

where,

$$m_D = \frac{2\pi K_w h(m_i - m)}{q_{sc}\mu p_{sc}}$$

$$m_{wD} = \frac{2\pi K_w h\left(m_i - m_{wf}\right)}{q_{sc}\mu p_{sc}}$$

$$t_D = \frac{K_w t}{\nu' r_w^{d_f}},$$

$$V_D = \frac{V - V_i}{V_i}$$

$$V_{ED} = \frac{V_E - V_i}{V_i}$$

$$\omega = \frac{\phi_w \mu C_g}{\nu'}$$

$$\lambda = \frac{K_w \tau}{\nu' r_w^{d_f}}$$

$$\nu' = \phi_w \mu C_g + \frac{2\pi K_w h V_i T Z_i F_G}{q_{sc} T_{sc}}$$

$$\beta = d_f - \theta - 1$$

where m is the pseudo pressure, $Pa^2/mPa\cdot s$, m_i is the pseudo pressure under the initial reservoir pressure which is p_i $Pa^2/mPa\cdot s$, q_{sc} is the production in standard state, m^3/s, p_{sc} is the pressure in standard state, Pa, T_{sc} is the temperature in standard state, K, V is the Coalbed methane concentration of matrix block, m^3/m^3, V_i is the Coalbed methane concentration when the pressure is p_i, m^3/m^3, V_E is the Coalbed methane concentration in equilibrium, m^3/m^3, V_L is the ultimate absorbance, m^3/m^3, τ is adsorption time, s, C_g is gas compressibility, Pa^{-1}, μ is gas viscosity, $Pa\cdot s$, h is formation thickness, m, Z_i is the gas compressibility factor, F_G is shape factor of matrix block $1/m^2$.

The Langmuir isothermal adsorption equation is

$$V_E = \frac{V_L m}{m_L + m} \tag{3}$$

$$V_i = \frac{V_L m_i}{m_L + m_i} \tag{4}$$

where m_L is the pseudo pressure when the pressure is p_L, $Pa^2/mPa\cdot s$, p_L is the pressure when the adsorbance is a half of ultimate adsorbance.

The definite condition is

$$\left[C_D\frac{\partial m_{wD}}{\partial t_D} - r_D^{\beta}\frac{\partial m_D}{\partial r_D}\right]\Bigg|_{r_D=1} = 1 \tag{5}$$

$$m_{wD} = \left[m_D - S r_D^{\beta}\left(\frac{\partial m_D}{\partial r_D}\right)\right]\Bigg|_{r_D=1} \tag{6}$$

$$\begin{cases} m_D(r_D = R_{eD}) = 0(\text{constant pressure boundary}) \\ \dfrac{\partial m_D}{\partial r_D}(r_D = R_{eD}) = 0(\text{sealed boundary}) \end{cases} \tag{7}$$

where C_D is dimensionless well bore storage coefficient, S is skin factor, R_{eD} is the dimensionless radius of outer boundary.

Laplace transform is a commonly integral transform used in engineering mathematics. The Laplace transform of function $f(t)$ is

$$L[f(t)] = \bar{f}(z) = \int_0^{\infty} f(t)e^{-zt}dt$$

where z is Laplace variables.

By using the Laplace Transform, the Eqs. (1) and (2) can be expressed as

$$\frac{\partial^2 \overline{m}_D}{\partial r_D^2} + \frac{\beta}{r_D}\frac{\partial \overline{m}_D}{\partial r_D} = r_D^{\theta}\left[\omega z\overline{m}_D - (1-\omega)z\overline{V}_D\right] \tag{8}$$