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Buckling and Vibrations of Multi-directional Functionally Graded Circular Plate Resting on Elastic Foundation

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Abstract

The present paper deals with the axisymmetric vibrations of multi-directional functionally graded circular plate under uniform in-plane loads resting on elastic foundation. The mechanical properties of the plate material are assumed to vary in both radial and transverse directions. Generalized differential quadrature method has been employed to obtain the frequency equations from the differential equation governing the motion of such simply supported plates. The effect of volume fraction index, in-plane force parameter, foundation parameter, heterogeneity parameter and density parameter has been studied on the natural frequencies of vibration. The critical buckling loads have been computed. Three-dimensional modes shapes for first three natural frequencies have been presented.

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Keywords: Axisymmetric vibrations;buckling, multi-directional FGM; generalized differential quadrature method; elastic foundation

1. Introduction

Functionally graded materials (FGMs) are new generation composite materials which were first introduced by a group of scientists in Japan in 1984 [1] as a means of preparing thermal barrier materials. Since their initiation, these materials have experienced a rapid development and found many applications in various fields of engineering. In classic ceramic/metal FGMs, the ceramic phase offers thermal barrier effects and protects the metal from corrosion and oxidation and the FGM is toughened and strengthened by the metallic constituent.

Due to the wide application of FGM plates in various engineering application, the dynamic behavior of FGM plates of various geometries has been studied by researchers throughout the world using analytical/numerical

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methods. A particular integration method has been employed by Liu et al. [2] to study the free vibrations of rectangular FGM plates with in-plane material inhomogeneity. Ritz method has been applied by Uymaz et al. [3] for the vibration analyses of FGM rectangular plate with in-plane material inhomogeneity on the basis of five-degree-of-freedom shear deformable plate theory. Bessel functions have been used by Baferani et al. [4] to obtain the exact solutions for free-vibration analysis of FGM thin annular sector plates resting on elastic foundation. Efforts have been made by researchers to analyze the effect of in-plane loads on the vibration characteristics of plates of various geometries. Najafizadeh and Heydari [5] have given exact solution for buckling of FGM circular plates based on higher order shear deformation theory under uniform radial compression. Lal and Ahlawat [6] have applied differential transform method to analyze the buckling and free axisymmetric vibration of FGM circular plate. A differential transform approach for free vibration and modal stress analyses of two-directional functionally graded circular plates with restrained edges and resting on two-parameter elastic foundations has been developed by Alipour and Shariyat [7]. Keeping the above in view, this paper presents the axisymmetric vibrations of multi-directional FGM circular plate resting on elastic foundation under uniform in-plane stressing using classical plate theory. The material properties are assumed to vary in both radial and transverse directions. Generalized differential quadrature method has been used to solve the fourth order differential equation. Three-dimensional modes shapes for first three natural frequencies have been presented. A comparison of results with the existing literature has been given.

2. Mathematical Formulation

Consider an FGM circular plate of radius *a*, thickness *h*, mass density and subjected to uniform in-plane tensile force N_0,resting on elastic foundation of modulus k_f and referred to a cylindrical co-ordinate system (*R*, , *z*). *z* = 0 being the middle plane of the plate. The top and bottom surfaces are *z* = *+h/2* and *z* = *-h/2*, respectively. The line *R* = 0 is the axis of the plate. The equation of motion governing transverse axisymmetric vibration of the present model (Figure 1) is given by [8]

$$D\,w_{,RRRR}+\frac{2}{R}\left[D+R\,D_{,R}\right]w_{,RRR}+\frac{1}{R^2}\left[-D+R(2+\nu)\,D_{,R}+R^2(D_{,RR}-N_0)\right]w_{,RR}$$
$$+\frac{1}{R^3}\left[D-R\,D_{,R}+R^2(\nu\,D_{,RR}-N_0)\right]w_{,R}+k_f\,w+\rho\,h\,w_{,tt}=0 \quad (1) \tag{1}$$

where *w* is the transverse deflection, *D* the flexural rigidity and the Poisson's ratio and a comma followed by a suffix denotes the partial derivative with respect to that variable.
For a harmonic solution, the deflection w can be expressed as

$$w(R,t)=W(R)e^{i\omega t}, \tag{2}$$

where is the radian frequency. The Eq. (1) reduces to

$$DW_{,RRRR}+\frac{2}{R}\left[D+R\,D_{,R}\right]W_{,RRR}+\frac{1}{R^2}\left[-D+R(2+\nu)\,D_{,R}+R^2D_{,RR}\right]W_{,RR}$$
$$+\frac{1}{R^3}\left[D-R\,D_{,R}+R^2\,\nu\,D_{,RR}\right]W_{,R}-N_0\,W_{,RR}-\frac{N_0}{R}W_{,R}+k_f W-\rho\,h\,\omega^2\,W=0. \tag{3}$$

Assuming that the top and bottom surfaces of the plate are ceramic and metal-rich, respectively in which the variations for the Young's modulus *E*(*R*, *z*) and the density (*R*, *z*) in both radial and transverse directions are given by