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Characterization of Root-Mean-Square Acceleration Errors in Flexible Structures undergoing Vibration Testing

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Abstract

A methodology is described to evaluate the Root-Mean-Square (RMS) errors in acceleration response during the vibration testing of flexible structures like missile and rockets. Simulation studies are carried out on a free-free uniform beam to characterize the error estimations in the desired and achieved acceleration spectra at different locations of the structure. Equal and unequal desired spectra are also considered at different locations of the structure and individual error in the achieved spectrum, as well as overall error is computed. The simulation results are obtained for traversing a single input along the length of the structure with three control sensors at different locations. The variation of the force requirement at different locations of the control sensor and the normalized RMS error for each control sensor is computed. The investigations show that the force requirement is less when the control sensors are placed at the far ends of the structure. It is also observed that the force requirement is increased marginally for the increased number of control sensors, whereas the overall error between the desired and achieved spectrum is reduced with the increase in the number of control sensors.

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Keywords: Root-mean-square Error; Acceleration Error; Flexible Structures; Vibration Testing.

1. Introduction

Extreme dynamic conditions, which are random in nature, originating from the aerodynamic loads and thrust fluctuations are the most critical parameters in the design phase of aerospace equipment, payloads, secondary structures and interfaces. The performance of the sub-systems depends on their operation under these extreme conditions. In the initial development phase of an aerospace structure, the random vibration levels experienced by the critical equipment are measured using an accelerometer for a certain number of flight configurations. Based on the measured acceleration levels, the acceleration spectrum is enveloped for all the flight configurations at the critical locations of the equipment. Each sub-system is subjected to vibration testing so that the functioning of the sub-systems can be evaluated for the simulated extreme vibration levels. In vibration testing [1-4], an airframe section assembled with electronic sub-systems is fixed to a shake table. A control accelerometer is placed on the shake table and a desired input acceleration spectrum for a specified duration is fed to the controller. The force to the airframe section is con-

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trolled in such a way that the desired acceleration spectrum is achieved at the base of the airframe section. Moreover, the input forces excite the structural frequencies of the test section and it becomes difficult to control the vibrations to the desired levels on a flexible structure. Hence, at present electronic systems are tested for their performance by mounting them on very rigid fixtures. The same procedure is adopted for all the sub-systems. The input acceleration spectrum and the duration of the vibration test differ from component to component. This demands for more vibration testing time for all the sub-systems and in turn delays the mission. Hence, the disadvantages of vibration qualification testing at section level are more time consumption, over testing and reduced life. In addition to this, the contribution of the flexibility of the overall structure on the vibration levels is not considered. However, the present scenario does not emulate the exact free-free boundary conditions of the aerospace structures during the flight. The authors [5-7] presented a methodology for vibration testing of a completely integrated aerospace vehicle under free-free boundary condition and is validated through the experiments. In this paper, an error estimation procedure is presented in the root-mean square values of the achieved and desired acceleration for the flexible structures.

2. General Formulation for Error Estimation

The equation of motion of a discrete structural system can be written as,

$$[M]\{\ddot{y}(t)\} + [C_d]\{\dot{y}(t)\} + [K]\{y(t)\} = [F]\{u(t)\} \tag{1}$$

where, $\{y(t)\} \rightarrow$ Displacement vector $= \{\, w_b(t)\ \theta(t)\,\}^T$ and $\{u(t)\} \rightarrow$ Force vector. From modal analysis approach [8,9], the response vector, $y(x,t)$ is given as,

$$\{y(x,t)\} = [P(x)]\{q_m(t)\} \tag{2}$$

where $[P(x)]$ is the modal matrix and $\{q_m(t)\}$ is the modal response vector.

The modal matrix and other modal characteristics can be obtained by solving the eigenvalue problem as,

$$\{[K] - [M]\omega^2\}\{\phi\} = \{0\};\ \ [P] = \left[\, \{\phi\}_1\ \{\phi\}_2\ \cdots\ \{\phi\}_N \,\right] \tag{3}$$

Using Eq. (2), Eq. (1) can be rewritten as,

$$[M][P]\{\ddot{q}_m(t)\} + [C_d][P]\{\dot{q}_m(t)\} + [K][P]\{q_m(t)\} = [F]\{u(t)\} \tag{4}$$

Pre-multiplying the above Eq. (4) by $[P]^T$, we get

$$[P]^T[M][P]\{\ddot{q}_m(t)\} + [P]^T[C_d][P]\{\dot{q}_m(t)\} + [P]^T[K][P]\{q_m(t)\} = [P]^T[F]\{u(t)\} \tag{5}$$

$$[M_m]\{\ddot{q}_m(t)\} + [C_m]\{\dot{q}_m(t)\} + [K_m]\{q_m(t)\} = [F_m]\{u(t)\} \tag{6}$$

where, $[M_m] \rightarrow$ Modal mass matrix, $[C_m] \rightarrow$ Modal damping matrix, $[K_m] \rightarrow$ Modal stiffness matrix, $[F_m] \rightarrow$ Modal force matrix

If we define the state vectors as,

$$\{z_1(t)\} = \{q_m(t)\};\ \{z_2(t)\} = \{\dot{q}_m(t)\} \tag{7}$$

Equation (6) can be rewritten in terms of the above defined state variables as,

$$\begin{Bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{Bmatrix} = \begin{bmatrix} [0] & [I] \\ -[M_m]^{-1}[K_m] & -[M_m]^{-1}[C_m] \end{bmatrix} \begin{Bmatrix} z_1(t) \\ z_2(t) \end{Bmatrix} + \begin{Bmatrix} [0] \\ [F_m] \end{Bmatrix} \{u(t)\} \tag{8}$$

$$\{\dot{z}(t)\} = [A]\{z(t)\} + [B]\{u(t)\} \tag{9}$$

where, $[A] \rightarrow$ State matrix, $[B] \rightarrow$ Input matrix

Considering the acceleration response, we can write the modal acceleration term $\ddot{q}_m(t)$, in terms of the state vector $z(t)$ and input vector $u(t)$ as,

$$\{\ddot{q}_m(t)\} = [C]\{z(t)\} + [D]\{u(t)\} \tag{10}$$