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Multivariate Extreme Value Distributions for Vector of Non-stationary Gaussian Processes

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Abstract

The focus of this study is on estimating the multivariate extreme value distributions associated with a vector of mutually correlated non-stationary Gaussian processes. This involves computing the joint crossing statistics of the vector processes by assuming the crossings to be Poisson counting processes. A mathematical artifice is adopted to take into account the dependencies that exist between the crossings of the processes. The crux in the formulation lies in the evaluation of a four-dimensional integral, which can be computationally expensive. This difficulty is bypassed by using saddlepoint approximation to reduce the dimension of the integral to be numerically computed to just two. The developments are illustrated through a numerical example and are validated using Monte Carlo simulations.

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Keywords: Multivariate extreme value distribution; Non-stationary Gaussian processes; Poisson counting process; Saddlepoint approximation

1. Introduction

Time variant reliability analysis of structural systems is usually studied in the time invariant format by defining the problem in terms of random variables that represent the extreme values of the response within a specified time interval. The focus therefore is on estimating the extreme value distributions of the response. For structural systems where the response constitutes a vector of correlated processes, estimates of the system reliability can be obtained from the knowledge of the joint multivariate extreme value distributions of the vector processes.

The problem of extreme value distributions for a vector of stationary Gaussian processes was earlier studied in [1]. This involved approximating the multivariate counting process associated with the level crossings as a multivariate Poisson random process. The successful development of the formulation required the evaluation of a six-dimensional integral, which was shown to be reduced to a two dimensional one using simplifying operations. This double integral was numerically evaluated and approximations for the multivariate extreme value distributions were obtained. Efforts to extend this methodology for non-stationary vector Gaussian processes was not possible as the non-stationary nature

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of the processes did not afford reduction in the dimension of the integrals [2] and as a result, the analytical formulation was not numerically viable. The present study byapsses these difficulties by taking advantage of the saddlepoint approximation method [3] to bring about a reduction in the dimension of the integrals, making the analytical approach for estimating the joint crossing statistics, computationally efficient even for non-stationary vector Gaussian processes.

2. Problem Statement

Consider $\{X_i(t)\}_{i=1}^k$ to be a vector of correlated non-stationary Gaussian random processes which are expressible in the form,

$$X_i(t) = e_i(t)X_{is}(t), \tag{1}$$

where, $X_{is}(t)$ is a stationary Gaussian random process and $e_i(t)$ is a deterministic envelope function, of the form

$$e_i(t) = a_i[\exp(-b_i t) - \exp(-c_i t)]. \tag{2}$$

Here, the parameters b_i and c_i determine the shape of $e_i(t)$ and a_i is a normalization factor such that $\max[e_i(t)] = 1.0$. The components $X_{is}(t)$ are assumed to be mutually correlated stationary Gaussian processes, whose spectral properties are defined in terms of the power spectral density (PSD) matrix $\mathbf{S}(\omega)$ or the covariance matrix $\mathbf{R}(\tau)$. For each $X_i(t)$, let us define $N_i(\alpha_i, 0, T)$ to be the number of upcrossings of level α_i in the time interval $[0, T]$ and $X_{m_i} = \max_{0\leq t\leq T} X_i(t)$ is defined to be the maxima of $X_i(t)$ in the time interval $[0, T]$. For a given i, $N_i(\alpha_i, 0, T)$ and X_{m_i} are random variables.

For scalar processes where $X_i(t)$ are Gaussian, it has been shown that X_{m_i} follows Gumbel distributions. The problem of estimating the joint probability distribution function (PDF) of $\mathbf{X_m} = \{X_{mi}\}_{i=1}^k$, when $\{X_i(t)\}_{i=1}^k$ constitute vector correlated stationary Gaussian processes has been discussed in [1]. For the sake of completion, the salient steps of the formulation are explained in sections 3 and 4.

3. Level Upcrossings

As previously defined, $\{N_i(\alpha_i, 0, T)\}_{i=1}^k$ is assumed to be a vector of multivariate Poisson random variables. For simplicity, let us consider the case of a bivariate process and define three mutually independent Poisson random variables U_1, U_2 and U_3, and λ_1, λ_2 and λ_3, respectively represent their respective parameters. Introducing the following transformations,

$$\begin{aligned} N_1(\alpha_1, 0, T) &= U_1 + U_3, \\ N_2(\alpha_2, 0, T) &= U_2 + U_3, \end{aligned} \tag{3}$$

it can be shown that, $N_1(\alpha_1, 0, T)$ and $N_2(\alpha_2, 0, T)$ are Poisson random variables with parameters $(\lambda_1+\lambda_3)$ and $(\lambda_2+\lambda_3)$, respectively. It can be further shown that λ_3 is the covariance of $N_1(\alpha_1, 0, T)$ and $N_2(\alpha_2, 0, T)$. Based on this, we can write

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{Bmatrix} = \begin{Bmatrix} \langle N_1(\alpha_1, 0, T)\rangle \\ \langle N_2(\alpha_2, 0, T)\rangle \\ \mathrm{Cov}\,[N_1(\alpha_1, 0, T), N_2(\alpha_2, 0, T)] \end{Bmatrix}, \tag{4}$$

where, $\langle\cdot\rangle$ denotes the mathematical expectation operator and Cov $[\cdot]$ denotes the covariance function. The expectation of the counting process, $\langle N_i(\alpha_i, 0, T)\rangle$ can be computed by integrating the mean upcrossing intensity with respect to time. The details of this can be found in section 5.

The covariance of N_1 and N_2 can be expressed as

$$\mathrm{Cov}[N_1, N_2] = \langle N_1(\alpha_1, 0, T)N_2(\alpha_2, 0, T)\rangle - \langle N_1(\alpha_1, 0, T)\rangle\langle N_2(\alpha_2, 0, T)\rangle. \tag{5}$$

The evaluation of joint expectation $\langle N_1(\alpha_1, 0, T)N_2(\alpha_2, 0, T)\rangle$ is central to this study since it involves evaluation of four dimensional integration which can be computationally demanding. A detailed discussion about evaluation of joint expectation is presented sections 6 and 7.