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ScienceDirect

Procedia Engineering 144 (2016) 512 – 519

Procedia Engineering

www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Material Parameter Identification in Transient Dynamics by Error in Constitutive Equation Approach

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Abstract

A study on material parameter identification of linearly elastic structures is presented in this work by minimizing the norm of the error in constitutive equation (ECE) from partial and corrupted measurements in transient dynamics. The identification problem is formulated as an optimization problem where the objective function measures the constitutive discrepancy due to the incompatible pair of stress and strain fields. These two fields are generated by solving two different forward problems related to linear elasticity. In the inverse algorithm, we used an effective penalty based approach to weakly satisfy the measured partial strain or displacement data. This technique not only allows us to incorporate the measurement field but helps to regularize the ill-posedness of the inverse problem. Here, we have proposed explicit material parameter update formulas for linear elastic materials. Eventually numerical examples of reconstruction of Young's modulus for 1D bar and beam are given here to present application of the proposed algorithm.

Peer-review under responsibility of the organizing committee of ICOVP 2015

Keywords: Identification problem; transient dynamics; linear elasticity; constitutive relation error.

1. Introduction

Material Parameter estimation of structures & damage detection from dynamic response data has enjoyed considerable attention in recent years. These techniques are used in different fields such as damage detection, structural health monitoring, seismic exploration, biomechanical imaging and in other applications. The time-domain identification schemes are an attractive alternative to overcome the drawbacks of the frequency domain and to yield more meaningful identification results.

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Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.035

Hjelmstad et al. [1] developed an estimation technique using recursive quadratic programming method (RQP) that can handle incompletely measured response in time, state, and space for complex structural systems. Kang et al. [2] used system identification scheme in time domain to estimate stiffness and damping parameters of a structure using least-squared error minimization between measured and calculated accelerations. Park et al. [3] developed a moving time window technique in system identification scheme for detection of sudden structural damage during an earthquake. The stiffness properties and the coefficients of Rayleigh damping have been considered as system parameters in this work. A useful review on the elastic parameter estimation methods has been given by Bonnet and Constantinescu [4]. Okafor and dutta [5] used wavelet transformation technique to detect the position and magnitude of damage (stiffness degradation) of aluminium cantilevered beam.

System identification problems are basically nonlinear constrained or unconstrained optimization problems. Optimum solution is found out by minimizing the difference or error between the calculated and measured response. Generally, second norm of error (L2 norm) is considered for the cost function of the optimization problem. Gauss-Newton method or quasi Newton methods (Oberai et al. [6]) are can be used for this purpose. Quasi Newton methods are used to solve large problems because of its less computational cost compared to Gauss- Newton methods, although the accuracy is less, especially in case of noisy data. The main disadvantages of L2 minimization error functional are its non-convexity and sensitivity to initial guess. Recently, a new technique for elastic parameter estimation is developed based on the concept of error in constitutive equation (ECE). The ECE based cost functional was first suggested by Ladèze and Leguillon [7]. Allix et al. [8] proposed a solution technique for material parameter identification by using the ECE based method from dynamic test data with high perturbations. ECE based technique has been explored in transient dynamic problem by Fiessel and Allix [9]. Recently, Banerjee et al. [10] developed a new MECE based method for identification of constitutive parameter for large system in frequency domain dynamics.

In this present study, an ECE based identification approach for elastic material parameter is proposed by minimizing the norm of the constitutive relation error resulting from incompatible stress and strain field in transient dynamics problem. We have also explored the Gauss-Newton method for comparison study with ECE based identification technique. For numerical experimentations, we have taken up the material parameter identification problem of 1D bar and beam.

2. Theoretical formulation

2.1. Forward problem

In this section, we are describing the forward problem of elastic body for transient dynamics. Let us consider $\bar{\Omega} = \Omega \cup \partial\Omega \subset \Re^d$ $(1 \le d \le 3)$ denotes a bounded and connected body. The fundamental balance equation of the dynamics of the deformable bodies can be written in the strong form as

$$\Delta.\boldsymbol{\sigma} + \mathbf{b} = -\rho\ddot{\mathbf{u}} \quad \text{in} \quad \Omega \tag{1}$$

$$\mathbf{u} = \mathbf{u_0} \quad \text{on} \Gamma_u \tag{2}$$

$$\boldsymbol{\sigma}.\mathbf{n_s} = \mathbf{t} \quad \text{on} \quad \Gamma_t \tag{3}$$

$$\boldsymbol{\sigma} = \mathrm{C} : \boldsymbol{\varepsilon} \tag{4}$$

$$\boldsymbol{\varepsilon}\left[\mathbf{u}\right] = \frac{1}{2}\left(\Delta\mathbf{u} + \Delta\mathbf{u}^{\mathrm{T}}\right) \tag{5}$$

Where, $\boldsymbol{\sigma}$ denotes the stress tensor, $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{b}$ is the body force, $\boldsymbol{t}$ is the applied traction force, $\boldsymbol{u}$ denotes the displacement field, $\boldsymbol{u}_0$ denotes essential boundary condition, $\boldsymbol{n}_s$ denotes outward unit normal. Γ_t and Γ_u respectively represents traction (natural) and displacement (essential) boundary condition. C is fourth-order linear elastic constitutive tensor. Also $\Gamma_t \cup \Gamma_u = \partial\Omega$, $\Gamma_t \cap \Gamma_u = \Phi$ where, $\partial\Omega$ denotes the total boundary and Φ denotes null set.