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Review on Thermo-mechanical Approach in the Modelling of Geo-materials Incorporating Non-Associated Flow Rules

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Abstract

Recently, there has been a burgeoning interest in developing constitutive soil models from the laws of thermodynamics, mainly due to the benefits that these models automatically obey them and the approach provides a well-established structure and reduces the need for 'ad hoc' postulates. A thermodynamic framework, also known as thermo-mechanical framework, has the capability to predict the behaviour of geotechnical materials, which requires the anticipated incorporation of non-associated flow rules. As it is very challenging to achieve acceptable accuracy in plasticity modelling of granular materials, this paper aims to review this framework not only to discuss the details of the major components but also to highlight the capability of generating non-associated flow rules in a natural way from thermo-mechanical principles. This approach introduces the use of internal variables to develop the two thermodynamic potentials (the free energy and the rate of dissipation functions), sufficient to derive the corresponding yield function, flow rule, isotropic and kinematic hardening rules as well as the basic elasticity law. It is shown that the non-associated flow rule can be derived naturally from the postulated stress-dependent dissipation increment function. Comparison has been made with stress-independent dissipation to demonstrate that the approach can also successfully explain the behaviour of standard materials with associated flow rules. The basic steps for the thermo-mechanical formulation for developing a constitutive model are also reviewed and summarised. Furthermore, the power of conventional mathematical technique, Legendre transformation, in the derivation of constitutive equations has been highlighted.

Keywords: thermo-mechanical; free energy; dissipation; frozen elastic energy; geo-materials; granular.

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1 Introduction

During the last decade, there has been a growing emphasis on the compliance of the developed constitutive soil models with the laws of thermodynamics, particularly the first and second laws. It was found out that the original Cam-clay model failed to invoke the second law of thermodynamics and hence, ultimately, led to its violation as a consequence. As a result, the modification of the Cam-clay model, i.e. modified Cam-clay model, abandoned the frictional mechanisms of energy dissipation in order to overcome the shortcomings of the original model and comply with the laws of thermodynamics. However, the modified Cam-clay model has not been able to incorporate non-associated flow rules, thus not capable to predict the behaviour of frictional geo-materials.

To be able to predict the behaviour of sands, rocks and other frictional geo-materials, it is now accepted that the constitutive modelling of such materials must include non-associated flow rules by formulating yield conditions and flow rules separately. Non-associated flow rules occur due to the interactions between the properties of frictional geo-materials.

Thermo-mechanical principles have been successfully applied to derive the yield condition and the flow rule from the dissipation function for the behaviour of standard materials with associated flow rules. However, non-associated flow rules can also be produced by following the thermo-mechanical approach, as suggested by (Collins & Houlsby, 1997). Therefore, a comprehensive review has been carried out to emphasise on the fact that if the dissipation function depends on the effective stress, as in the case of modelling dissipative geo-materials, non-associated flow rules can be generated automatically from thermo-mechanical principles. It is also discussed that the use of traditional mathematical procedure, termed “Legendre transformation”, plays a vital role in this approach.

2 Thermo-mechanical Framework

The most appropriate starting point for an isothermal deformation of materials can be written as:

$$\delta W = d\Psi + \delta\Phi, where\ \delta\Phi \geq 0 \tag{1}$$

where δW is the incremental effective work done on a continuum element, $d\Psi$ is the differential of the free energy defined per unit volume and $\delta\Phi$ is the dissipation increment function defined per unit volume. $\delta\Phi$ must be non-negative in order to comply with the second law of thermodynamics. The free energy is a function of state variables, e.g. elastic and plastic strains. In contrast, Φ and W are not state functions. The free energy is assumed to be a function of total elastic strain tensor and plastic strain tensor, $e_{ij}\ and\ e_{ij}^{p}$, respectively. On the other hand, the dissipation function is assumed to depend additionally on plastic strain rate tensor, i.e., $\delta\Phi\left(e_{ij}, e_{ij}^{p}, de_{ij}^{p}\right)$. The validity of these assumptions can be found in (Collins & Houlsby, 1997) and (Collins & Kelly, 2002).

In general, the free energy is allowed to depend on both the elastic and plastic strains. This requires additional assumption, in which the material must be ‘decoupled’ in the sense that the instantaneous elastic moduli do not depend on the plastic strains. The special case comes from the above assumption that the free energy can be expressed as the sum of a function of only elastic strains, plus a function of only plastic strains. The validity of these assumptions can be found in (Collins & Houlsby, 1997) and (Collins & Kelly, 2002). From this ‘decoupled’ assumption, it follows that:

$$\Psi = \Psi^{e}\left(e_{ij}^{e}\right) + \Psi^{p}\left(e_{v}^{p}\right) \tag{2}$$