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Application of the Upper and Lower-Bound Theorems to Three-Dimensional Stability of Slopes

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Abstract

A finite element software implementing both the upper and the lower-bound theorems of limit analysis has been recently developed in the Civil Engineering Department of FCT/UNL. This tool has the ability to automatically determine compatible velocity fields for the upper-bound case and equilibrated and plastically admissible stress fields for the lower-bound case. The upper and lower bounds are obtained by a common iterative solution scheme. The latter is suitable for parallelization, allowing large three-dimensional problems to be solved more rapidly. These tools can therefore be applied to three-dimensional geotechnical problems such as 3D slope stability analyses. The paper will present a brief description of the algorithm and its application to three-dimensional slopes with different geometries and soil properties.

Keywords: Limit Analysis, Three-Dimensional Slope Stability, Parallelization, Finite Elements

1 Introduction

Slope stability is a Soil Mechanics classical problem that has been studied by numerous authors in two-dimensional conditions (2D), using limit equilibrium methods [6, 17, 18, 9, 2, 14, 16, 8, 5]. Within the framework of limit analysis, a few authors presented some results for the same 2D conditions [4, 21, 10]. For three-dimensional (3D) conditions, however, very few contributions have been made using limit analysis. A compilation of contributions of such approach can be found in [7]. More recently in [11] the authors used upper and lower bound numerical analyses to propose stability charts for 3D undrained slopes. In [13, 12] the authors used upper-bound analytical methods for both undrained and drained 3D slopes. In the knowledge of the authors there are no lower bound solutions published in the literature for drained 3D slope conditions. The present paper uses an upper and lower bound numerical tool, *Mechpy*, in the analysis of 3D slopes in drained conditions with the aim to improve the available solutions and to present

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a narrow gap between the two approaches of limit analysis, resulting, for practical purposes, in the exact solution.

Mechpy is an in-house software implemented in Python language for the development of non-conventional finite element formulation. The limit analysis module of this software is an evolution of *sublim3D* [1, 19, 20]. It uses the Alternating Direction Method of Multipliers (ADMM) technique. Its iterative solution scheme is based on an operator splitting algorithm, which is suitable to efficiently solve large-scale variational problems with parallel processing. This allows using fine 3D finite element meshes and obtaining highly accurate solutions.

2 Numerical Formulation

As it is usual in limit analysis, an optimization problem is formulated. In this paper, two complementary finite element analysis formulations, based on the kinematical and the static theorems, are used in order to determine an upper and a lower bound of the true collapse load multiplier of a mechanical structure, respectively. These kind of problems, in general, have a significant number of decision variables and constraints. To obtain its solutions the ADMM algorithm has been used by the authors, because it is a very versatile and robust (practically never fails to converge) technique and is also inherently parallelizable [3].

2.1 The Discrete Limit Analysis Problem

According to [20], the discrete limit analysis problem consist in finding a collapse load multiplier, α ($\alpha \in \mathbb{R}^+$), in order to minimize:

$$\alpha(\mathbf{v}, \mathbf{e}, \lambda) = W_{\mathcal{D}}(\mathbf{e}) - \mathbf{F}_0^T \mathbf{v} + \lambda(1 - \mathbf{F}^T \mathbf{v}) \tag{1}$$

subject to:

$$\mathbf{B}\mathbf{v} - \mathbf{e} = \mathbf{0} \tag{2}$$

with,

$$W_{\mathcal{D}}(\mathbf{e}) = \begin{cases} \int_\Omega \mathcal{D}(\mathbf{e}_j) \mathrm{d}\Omega & \mathbf{e}_j \in \mathcal{C}_c, \text{ for all gauss points } j \\ +\infty & \text{otherwise} \end{cases} \tag{3}$$

where $\mathbf{B}$ is the discrete compatibility operator, $\mathbf{v}$ is the nodal velocity vector and $\mathbf{e}$ collects all the strain rate components at the gauss points. $\mathbf{F}$ and $\mathbf{F}_0$ denote the nodal force vectors of the live and dead loads, respectively. The latter are not affected by the load multiplier, α. Lastly, λ is a Lagrange multiplier, devoid of any physical meaning. In (3), $W_{\mathcal{D}}$ is the total dissipated energy within volume Ω, $\mathcal{D}$ is the plastic energy dissipation rate per unit volume (expressed in terms of kinematic fields only [15]) and $\mathcal{C}_c$ is the space formed by all plastically admissible strain states (orthogonal to the yielding surface in at least one point [15], thus imposing an associated flow rule).

Note that (1) is the form that naturally derives from direct discretizations of the upper bound theorem. However, based on the principles of optimization duality, discrete lower bound formulations may be cast in an identical form as well, provided that the appropriate discrete operators and approximation functions for the velocity field are adopted.

2.2 Outline of the Solution Procedure

The ADMM algorithm [3] is used to find the optimal solution of the discrete limit analysis problem (1). This scheme, as shown in Table 1, is a cyclic k iteration with a two steps minimization procedure followed by a third stage where the Lagrange multipliers, $\mathbf{s}$, are updated.