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Heat Transfer in Binary Gas Mixtures Confined in a Lid-Driven Square Cavity

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Abstract

Grad's moment method is employed to investigate heat transfer in binary gaseous mixtures confined in a lid-driven square cavity numerically via finite difference method. Preliminary results on heat transfer are presented. Complex heat flow phenomenon of anti-Fourier heat transfer is revealed by moment equations which Navier–Stokes and Fourier equations fail to capture.

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Keywords: Boltzmann equation; Grad's moment method; lid-driven cavity; gas mixtures; transition regime.

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1. Introduction

Owing to advent of miniaturization, fluid flows at micro- and nano-scales have received considerable attention in the last few years. For designing, fabricating and optimizing performances of micro-devices, a good understanding of heat transfer mechanism in them is essential.

It is well-known that the Navier–Stokes and Fourier (NSF) equations break down in describing rarefied gaseous flows, particularly in the so-called transition regime and beyond [1–5]. The rarefaction of a gas can be characterized by the Knudsen number (Kn), which is defined as the ratio of mean-free-path of the gas to a characteristic length scale of the device. The rarefied flow conditions that occur in micro-devices are generally caused by their extremely small sizes because owing to their small sizes, the mean-free-path of the gas confined in them becomes comparable with their geometric dimensions—even at standard atmospheric conditions—and, thereby, the Knudsen number in micro-devices is not very small. The Knudsen number in micro-devices, typically, ranges from 0.01 to 1 [3,6], which is a combination of the range of the slip-flow regime ($0.001 \lesssim \mathrm{Kn} \lesssim 0.1$) and that of the transition regime ($0.1 \lesssim \mathrm{Kn} \lesssim 10$), see e.g., [1,7–10]. In general, the NSF equations start loosing their validity already in the slip-flow regime; nonetheless, they are still acceptable for describing processes in the slip-flow regime if supplemented with appropriate velocity slip and temperature jump boundary conditions [11,12]. However, for processes in the transition regime, the NSF equations fail completely and their use is not recommended in this regime [1]. Additionally, particle-based methods, such as direct simulation Monte Carlo (DSMC) [13] and discrete velocity methods [14], are

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computationally very expensive for processes in the transition regime due to large number of molecules. Therefore, for describing processes in the transition regime, more refined models are required. These models, usually, emanate from the Boltzmann equation [15], which is the fundamental equation in the kinetic theory of gases and is adept for describing processes in all regimes (i.e., for all Knudsen numbers) but, unfortunately, extremely difficult to solve analytically and very expensive numerically. The most famous and widely used extended macroscopic models are Grad's 13-moment (G13) equations [16] and its variant—the regularized 13-moment (R13) equations [1,17,18] (see also [19]). These models are based on Grad's method of moments [16] and are known to capture several non-equilibrium effects in the early transition regime with a reasonable compromise between numerical accuracy and computational cost.

Heat transfer in a lid-driven square cavity is a classical problem of fluid dynamics which has been studied extensively in the context of Newtonian fluids, see e.g., [20] and references therein, and by a few authors in context of single rarefied gases, see e.g., [21–23]. However, the same problem has received much less attention in the context of rarefied gaseous mixtures.

In this paper, we consider the problem of heat transfer in binary mixtures of monatomic-inert-ideal (noble) gases confined in a square cavity whose top wall (lid) is moving with a constant velocity while all other walls are stationary and study it through the Grad's 2×13-moment (2×G13) equations for binary mixtures of noble gases interacting with Maxwell interaction potential (or made up of Maxwell molecules (MM)) numerically via finite difference method.

The rest of the paper is organized as follows. The problem and linearized Grad's moment equations for binary gas mixtures along with required boundary conditions are described in § 2. Preliminary results for the lid-driven cavity problem are presented in § 3. The paper ends with a conclusion in § 4.

2. Problem description and method of solution

2.1. Problem description

We consider a binary mixture of gases α and β—composed of MM—confined in a square cavity of side length L. Let the temperatures of all the walls of the cavity be same and equal to a constant value $T_w = T_\circ + \varepsilon \tilde{T}_0$; moreover, let the top wall (lid) of the cavity be moving in positive x-direction with velocity $v_{\text{lid}} = \varepsilon \tilde{v}_0$. The schematic of the lid-driven square cavity is shown in Fig. 1. Here, ε is a smallness parameter, which is also used in linearizing the moment equations and boundary conditions. The third dimension z of the cavity is assumed very long so that the flow takes place essentially in two dimensions (x and y) and thus, z-axis in Fig. 1 is just for illustration purposes. We are interested in studying the heat transfer in steady state.

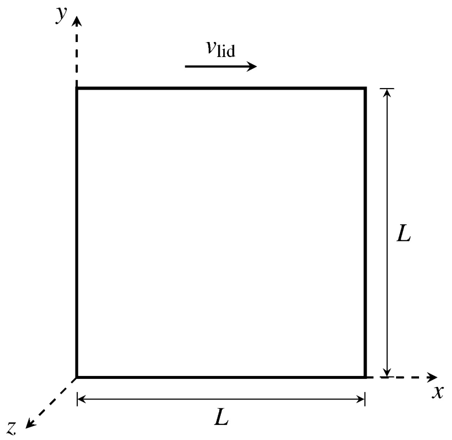

Fig. 1. Schematic of a two-dimensional lid-driven square cavity; z-axis is included just for illustration purposes.