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Exponentially Fitted Initial Value Technique for Singularly Perturbed Differential-Difference Equations

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Abstract

In this paper, an exponentially fitted initial value technique is presented for solving singularly perturbed differential-difference equations with delay as well as advance terms whose solutions exhibit boundary layer on one (left/right) of the interval. It is distinguished by the following fact that the original second order differential-difference equation is replaced by an asymptotically equivalent singular perturbation problem and in turn the singular perturbation problem is replaced by an asymptotically equivalent first order problem and solved as an initial value problem using exponential fitting factor. To validate the method, model examples with boundary layers have been solved by taking different values for the delay parameter δ, advance parameter η and the perturbation parameter ε. The effect of the small shifts on the boundary layer has been investigated and presented in graphs. Theoretical convergence of the scheme has also been investigated.

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1. Introduction

A differential-difference equation model incorporating stochastic effects due to neuronal variability was represented by Stein [11] and the solution to this model was approximated by Monte Carlo techniques. More generalization of this model, to deal with distribution of post synaptic potential amplitudes, was discussed by Stein [12]. Asymptotic approach to study general boundary-value problems for singularly perturbed differential-difference equations was given in a series of papers by Lange and Miura [9,10]. A variety of numerical approaches have been presented by Kadalbajoo and Sharma [7,8] for singularly perturbed differential-difference equations with only negative shift and as well as with both positive and negative shifts. Mirzaee and Hoseini [3] presented an approach with collocation method and matrices of Fibonacci polynomials to solve differential-difference equations with negative and positive shifts. Xu and Jin [5] constructed the formula of asymptotic expansion, using boundary function and fractional steps, to study vector singular perturbed delay differential equations. Genga and et al.[4] discussed numerical treatment of singularly

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perturbed differential-difference equations, by reproducing kernel method.
Pertaining to the above literature, in this paper a numerical technique, namely an exponentially fitted initial value technique has been presented for solving singularly perturbed differential-difference equations with delay as well as advance terms. The second order differential-difference equation is replaced by an asymptotically equivalent singular perturbation problem and in turn the singular perturbation problem is replaced by an asymptotically equivalent first order which is solved using exponential fitting factor.

2. Initial Value Technique

We consider singularly perturbed differential-difference equation of the form:

$$\varepsilon y''(x) + a(x)y'(x) + b(x)y(x-\delta) + d(x)y(x) + c(x)y(x+\eta) = r(x)\ ;\ 0 < x < 1 \tag{1}$$

subject to the interval and boundary conditions

$$y(x) = \phi(x)\ ,\ \text{on } -\delta \le x \le 0 \tag{2}$$

$$y(x) = \gamma(x),\ \text{on } 1 \le x \le 1+\eta \tag{3}$$

where $a(x)$,$b(x)$,$c(x)$,$d(x)$,$r(x)$,$\phi(x)$ and $\gamma(x)$ are bounded and continuously differentiable functions on $(0, 1)$, $0 < \varepsilon << 1$ is the singular perturbation parameter; and $0 < \delta = o(\varepsilon)$ and $0 < \eta = o(\varepsilon)$ are the delay and the advance parameters respectively. In general, the solution of (1)-(3) exhibits boundary layer behavior of width $O(\varepsilon)$ for small values of ε. By using Taylor series expansion in the neighborhood of the point x, we have

$$y(x-\delta) \approx y(x) - \delta y'(x) \tag{4}$$

$$y(x+\eta) \approx y(x) + \eta y'(x) \tag{5}$$

By using Eqs. (4) and (5) in (1) we get an asymptotically equivalent second order singular perturbation problem of the form:

$$\varepsilon y''(x) + p(x)y'(x) + Q(x)y(x) = r(x) \tag{6}$$

with

$$y(0) = \phi(0) = \phi_0 \tag{7}$$

$$y(1) = \gamma(1) = \gamma_1 \tag{8}$$

where

$$p(x) = a(x) + c(x)\eta - b(x)\delta\ ;\ \ Q(x) = b(x) + c(x) + d(x)$$

Since $0 < \delta << 1$ and $0 < \eta << 1$, the transition from Eq. (1) to Eq. (6) is admitted (El'sgolt's and Norkin [2]) and the solution of Eq. (6) will provide a good approximation to the solution of Eq. (1).

2.1. Left End Boundary Layer Problems

We assume that $Q(x) \le 0$, $p(x) \ge M > 0$ throughout the interval $[0, 1]$, where M is some constant. Under these assumptions, Eq. (6) has a unique solution $y(x)$ which exhibits a boundary layer of width $O(\varepsilon)$ on the left side of the interval, i.e., at $x = 0$. For convenience, we shall write Eq. (6) as follows:

$$\varepsilon y''(x) + (p(x)y(x))' + q(x)y(x) = r(x)\ ;\ q(x) = Q(x) - p'(x) \tag{9}$$

with

$$y(0) = \phi(0) = \phi_0\ ;\ y(1) = \gamma(1) = \gamma_1 \tag{10}$$

Now, we extended the initial value technique developed by Kadalbajoo and Reddy [6], for a class of nonlinear singular perturbation problems, to solve the Eqs. (9)-(10). The resulting initial value problem is given as follows:

$$Ly \equiv \varepsilon y'(x) + p(x)y(x) = f(x) + K \tag{11}$$