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ScienceDirect

Procedia Engineering 119 (2015) 226 – 234

Procedia Engineering

www.elsevier.com/locate/procedia

13th Computer Control for Water Industry Conference, CCWI 2015

Study on the frequency response function of viscoelastic pipelines using a multi-element Kevin-Voigt model

Jinzhe Gong[a,]*, Aaron Zecchin[a], Martin Lambert[a] and Angus Simpson[a]

[a]*School of Civil, Envrionmental & Mining Engineering, the University of Adelaide, Adelaide, South Australia 5005, Australia*

Abstract

This research focuses on the study of the frequency response function (FRF) of viscoelastic pipelines. The transfer matrix of a uniform viscoelastic pipeline is derived using the generalized multi-element Kevin-Voigt (K-V) model. The analytical expressions of the characteristic impedance and propagation operator are presented. The frequency response diagrams (FRDs) of a viscoelastic pipeline, with or without unsteady friction, are obtained numerically and compared with the FRDs of an elastic pipeline, with or without unsteady friction. It is verified that the pipe wall viscoelasticity introduces not only frequency-dependent shifting of the resonant frequencies but also frequency-dependent damping of the resonant peaks.

Keywords: frequency domain, numerical simulation, plastic, transient, transfer matrix, water hammer

1. Introduction

The use of plastic pipelines, such as polyvinyl chloride (PVC) and high density polyethylene (HDPE) pipes, has been gradually increasing throughout the world for both portable water distribution and sewage transport. The understanding of the hydraulic behaviour of plastic pipelines under hydraulic transient (water hammer) events is important and necessary for better system design and safe operation. For plastic pipelines, one distinctive feature when compared with conventional metallic pipelines is the pipe wall viscoelasticity.

Over the last 40 years, a number of studies have been conducted on how the wall viscoelasticity affect the hydraulic transient behaviour of a pipeline. However, most studies were focused on the transient response of a viscoelastic pipeline in the time domain [1-5]. It is known that pipe wall viscoelasticity introduces both extra

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* Corresponding author. Tel.: +61-(0)8-8313-1231; fax: +61-(0)8-8313-4359.
E-mail address: jinzhe.gong@adelaide.edu.au

doi:10.1016/j.proeng.2015.08.880

dissipation (damping) and dispersion (delay or phase shift) to the transient pressure response of a pipeline in the time domain [6].

Frequency domain studies on the transient response of elastic pipelines are abundant and a number of frequency response function (FRF)-based pipeline fault detection techniques have been proposed for elastic pipelines [7-11]. In the contrast, studies on the FRF of viscoelastic pipeline are few and limited. Suo and Wylie [12] used complex-valued and frequency-dependent wave speeds to describe the viscoelastic behaviour of a pipeline. It was found that the use of complex wave speeds introduced shifting in the resonant frequencies, and vibration components at higher frequencies decay faster. Duan et al. [13] derived the transfer matrix of a viscoelastic pipeline, with and without a leak, using a one-element Kevin-Voigt (K-V) model. It was concluded that the pipe wall viscoelasticity would shift the resonant frequencies but would not visibly modify the leak-induced sinusoidal pattern in the resonant peaks, which implies that the viscoelastic effects on the amplitude of resonant responses is either uniform or negligible. However, a later review paper by Lee et al. [14] stated that a viscoelastic pipe would have a frequency-dependent attenuation in the resonant response, though no mathematical justification was provided in that paper.

This research aims to expend the knowledge of transient behaviour of viscoelastic pipelines in the frequency domain, in particular, to study the FRF of a viscoelastic pipeline. The generalized multi-element K-V model is used, which is more suitable to describe the viscoelastic phenomenon of polymer than the simplified one-element K-V model. The transfer matrix method [15] is used in the analysis and derivation. The transfer matrix of a viscoelastic pipeline is derived, in which the analytical expressions of the characteristic impedance and the propagation operator of a pipeline are described by the generalized multi-element K-V model. Numerical simulations are conducted to study the frequency response diagrams (FRDs) of a viscoelastic pipeline, with and without unsteady friction, in a *reservoir-pipeline-oscillating valve* configuration. The FRDs from a frictionless elastic pipeline and an elastic pipeline with unsteady friction are also simulated and compared. The results of this research verify that pipeline viscoelasticity can induce both non-uniform shifting in the resonant frequencies and significant frequency-dependent damping on the resonant peaks.

2. Time-domain governing equations for viscoelastic pipelines

This section reviews relevant time-domain water hammer equations for a viscoelastic pipeline. The momentum equation that describes one-dimensional (1-D) transient flow in pressurized pipelines is [15]

$$\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{\partial H}{\partial x}+h_f=0 \tag{1}$$

where g is gravitational acceleration, A is the cross-sectional area of a pipeline, Q is the flow rate, H is the piezometric head, t is time, x is distance along the pipeline, and h_f is the head loss per unit length due to friction. The head loss can be regarded as a summation of a steady-state component h_{fs} and an unsteady-state component h_{fu}, and written as

$$h_f=h_{fs}+h_{fu} \tag{2}$$

The steady-state component is well defined for both laminar and turbulent flow [15]. The expression for the unsteady-state component is still a topic of research and several unsteady friction formulas are reported in the literature [16].

The continuity equation for 1-D transient flow in pressurized pipelines, when linear viscoelastic behavior of the pipe wall is considered, is given as [1-3]

$$\frac{gA}{a_e^2}\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}+2A\frac{\partial \varepsilon_r}{\partial t}=0 \tag{3}$$