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Efficient preconditioned iterative methods for hydraulic simulation of large scale water distribution networks

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Abstract

In this paper, we consider the use of an efficient null space algorithm for hydraulic analysis that employs preconditioned conjugate gradient (PCG) methods for solving the Newton linear equations. Since large water network models are inherently badly conditioned, a Jacobian regularization is employed to improve the condition number to some degree, this resulting in an inexact Newton method whose analyses is presented. Based on this analysis, constraint preconditioners are used to improve the condition number further for more efficient use of CG solvers. Operational networks are used to study the computational properties of the various approaches.

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Keywords: Water distribution networks, hydraulic analysis, inexact Newton method, preconditioned conjugate gradient, null space algorithm

1. Introduction

The Newton method for hydraulic analysis has a Jacobian with a saddle point structure [1,2]. In the numerical optimization literature, null space algorithms for saddle point problems have been used extensively, often called *reduced Hessian* methods [2]. Null space algorithms, as opposed to the range space approach of GGA [3], have also been applied for hydraulic analysis of water and gas pipe networks [1,4–6]. For a WDN with n_p number of pipes (or links) and n_n unknown-head nodes, the number $n_l = n_p - n_n$, which is the number of co-tree flows [6], is often much smaller than n_n. At each iteration, whereas the GGA method solves a linear problem of size n_n, a null space method solves an often much smaller problem of size n_l but with the same symmetric positive definiteness properties. Therefore, significant computational savings can be made for sparse network models. Moreover, GGA becomes singular when one or more of the head losses vanish. Unlike the GGA approach, null space algorithms do not involve inversion of headloss values. As such, they will not require processes to deal with zero flows so long as there are no loops with all zero flows [1,2,6].

In this article, we analyze an efficient null space algorithm based Newton method for hydraulic analysis proposed in [1]. By using sparse null space basis, we show that a significant fraction of the network pipes need not be involved in the flow updates of the null space Newton method. Taking advantage of this, a partial update scheme is used to

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reduce the number of expensive computations in calculating head losses . Since the flow update equations of the null space algorithm do not depend on pressure evaluations, computing pressure heads near convergence will reduce the number of pressure head computations for further computational savings.

In this framework, the Newton steps are computed by solving linear equations projected in the much smaller dimensional kernel space of the flow conservation constraints. The resulting linear equations are sparse, symmetric and positive definite; therefore, sparse iterative methods are considered for solving them. However, these linear systems are inherently very badly conditioned due to the large scale of variation in pipe loss characteristics and flows when considering operational water network models. We consider the use of Jacobian regularization in the Newton method [7] to keep the condition number low. We show that the resulting method will be an inexact Newton method; we propose appropriate condition number bounds for the regularization that will not affect the convergence properties of the Newton method. In addition, we propose and study different tailored constraint preconditioners for use with the conjugate gradient (CG) method that will reduce the condition number further and enhance the rate of convergence of the CG iterations. We demonstrate through case studies which preconditioners are most effective.

2. Problem Formulation

In demand-driven hydraulic analysis, the demand is assumed known, as opposed to pressure-driven simulations where demands are written as functions of pressure [8] to be solved for. Once a WDN is defined by its connectivity, and the characteristic of its pipes and the demands at each node, a steady-state solution of the system is computed by solving the flow conservation and energy loss equations for a given demand. The objective is to compute the unknown flows in each pipe and the pressures at the demand nodes. Let pipe p_j have flow q_j going from node i to node k, and with pressure heads h_i and h_k at nodes i and k, respectively. The head loss across the pipe can then be represented as:

$$h_i - h_k = r_j |q_j|^{n-1} q_j, \tag{1}$$

where r_j, the resistance coefficient of the pipe, can be modelled as either independent of the flow or implicitly dependent on flow q_j and given as $r_j = \alpha L_j/(C_j^n D_j^m)$. The variables L_j, D_j and C_j denote the length, diameter and roughness coefficient of pipe j, respectively. The triplet α, n and m depend on the energy loss model used; Hazen-Williams (HW: $r_j = 10.670 L_j/(C_j^{1.852} D_j^{4.871})$) and Darcy-Weisbach (DW) are two commonly used loss formulae [7]. In DW models, the dependence of the resistance coefficient on flow is implicit; see the formulae in [9, (1–2)].

Given is a network with n_p links connecting $n_n (< n_p)$ unknown head nodes, and n_0 known head nodes. We define the vector of unknown flows and pressure heads as $q = [q_1, \ldots, q_{n_p}]^T$ and $h = [h_1, \ldots, h_{n_n}]^T$. With head loss equations defined for each pipe and the fixed heads and demands for each node taken into account, the set of nonlinear equations that define the steady state flow conditions are given by the matrix equation [10, Eq. (1)]:

$$f(q, h) := \begin{pmatrix} A_{11}(q) & A_{12} \\ A_{12}^T & 0 \end{pmatrix} \begin{pmatrix} q \\ h \end{pmatrix} + \begin{pmatrix} A_{10} h_0 \\ -d \end{pmatrix} = 0 \tag{2}$$

where the first and second block-row of equations represent the conservation of energy (pressure heads) and flow continuity laws, respectively, $h_0 \in \mathbb{R}^{n_0}$ and $d \in \mathbb{R}^{n_n}$ represent the known heads (eg. at a reservoir or tank) and demands at nodes, respectively. The matrix $A_{12}^T \in \mathbb{R}^{n_n \times n_p}$ is the incidence matrix for the n_n unknown head nodes. The square matrix $A_{11} \in \mathbb{R}^{n_p \times n_p}$ is a diagonal matrix with the elements

$$A_{11}(j, j) = r_j |q_j|^{n_j - 1}, \; j = 1, \ldots, n_p, \tag{3}$$

representing part of the loss formula in (1).

Most non-linear equations and unconstrained optimization problems are solved using Newton's method [11,12]. The same Newton method has been applied to solve the hydraulic analysis problem, for the first time in [10], and has been extensively used for the same purpose since then. By considering the Jacobian of $f(q, h)$ with respect to the unknown $x := [q \;\; h]^T$, and using the head loss model for the i^{th} link (3), the Newton iterations for the solution of (2)