Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 101 (2015) 69 – 76

Procedia Engineering

www.elsevier.com/locate/procedia

3rd International Conference on Material and Component Performance under Variable Amplitude Loading, VAL2015

Invariant-Based and Critical-Plane Rainflow Approaches for Fatigue Life Prediction under Multiaxial Variable Amplitude Loading

Marco Antonio Meggiolaro[a], Jaime Tupiassú Pinho de Castro[a,*], Hao Wu[b]

[a]*Dept. of Mechanical Engineering, Pontifical Catholic University of Rio de Janeiro, Brazil*
[b]*School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai, P.R. China*

Abstract

Multiaxial variable amplitude histories usually require a rainflow algorithm to identify individual cycles. A computationally-efficient 5D multiaxial rainflow algorithm that can deal with any 1D to 6D history has been proposed by the authors, based on the representation of 6D stresses and strains in 5D deviatoric sub-spaces that use a von Mises metric. These spaces provide simple geometric interpretations for the identified load cycles while highly reducing computational cost. In this work, the most efficient multiaxial rainflow algorithms for variable amplitude histories are presented and compared, detailing their individual advantages and disadvantages.

Peer-review under responsibility of the Czech Society for Mechanics

Keywords: Multiaxial rainflow; Multiaxial fatigue; Critical plane approach; Non-proportional loading.

1. Introduction

The identification of individual cycles of variable amplitude loading (VAL) histories is required for applying most fatigue damage models, but the traditional rainflow algorithm cannot be directly applied to multiaxial histories, since it can count only one stress or strain component. A possible approach to solve this problem is to project the multiaxial stress or strain VAL history onto a so-called candidate plane, where the crack is assumed to initiate at the critical point

* Corresponding author. Tel.: +55-21-35271642.
E-mail address: jtcastro@puc-rio.br

Peer-review under responsibility of the Czech Society for Mechanics
doi:10.1016/j.proeng.2015.02.010

of the structural component. The number of such planes can be narrowed down to a manageable size assuming that the fatigue failure mechanism is associated with a single dominant crack [1], as usual in most metallic alloys, and free surface conditions, i.e. $\tau_{xz} = \tau_{yz} = \mathbf{0}$ and $\gamma_{xz} = \gamma_{yz} = \mathbf{0}$, where $\mathbf{z}$ is perpendicular to the surface. The free-surface formulation can allow the presence of a compressive surface stress $\sigma_z = -\mathbf{p} \leq \mathbf{0}$ caused by a surface pressure $\mathbf{p} \geq \mathbf{0}$. Under these conditions, the stresses and strains projected onto a candidate plane rotated at (θ, ϕ) from the surface, as illustrated in Fig. 1, become

$$
\begin{aligned}
&\tau_A(\theta,\phi) = (\tau_{xy}\cos 2\theta + 0.5\cdot(\sigma_y - \sigma_x)\cdot\sin 2\theta)\cdot\sin\phi \\
&\tau_B(\theta,\phi) = 0.5\cdot(\sigma_x\cdot\cos^2\theta + \sigma_y\cdot\sin^2\theta + p + \tau_{xy}\sin 2\theta)\cdot\sin 2\phi \\
&\sigma_\perp(\theta,\phi) = (\sigma_x\cdot\cos^2\theta + \sigma_y\cdot\sin^2\theta + \tau_{xy}\sin 2\theta)\cdot\sin^2\phi - p\cdot\cos^2\phi \\
&\gamma_A(\theta,\phi) = (\gamma_{xy}\cos 2\theta + (\varepsilon_y - \varepsilon_x)\cdot\sin 2\theta)\cdot\sin\phi \\
&\gamma_B(\theta,\phi) = (\varepsilon_x\cdot\cos^2\theta + \varepsilon_y\cdot\sin^2\theta - \varepsilon_z + 0.5\cdot\gamma_{xy}\sin 2\theta)\cdot\sin 2\phi \\
&\varepsilon_\perp(\theta,\phi) = (\varepsilon_x\cdot\cos^2\theta + \varepsilon_y\cdot\sin^2\theta + 0.5\cdot\gamma_{xy}\sin 2\theta)\cdot\sin^2\phi + \varepsilon_z\cdot\cos^2\phi
\end{aligned}
\tag{1}
$$

where $\sigma_\perp$ and $\varepsilon_\perp$ are the normal stress and strain perpendicular to the candidate plane, τ_A and γ_A are the shear stress and strain parallel to the free surface, and τ_B and γ_B are the shear stress and strain in the depth direction, see Fig. 1, while σ_x, σ_y, τ_{xy}, ε_x, ε_y and γ_{xy} are the time-varying stress and strain components caused by the VAL history. Under such free-surface conditions, the expected fatigue cracks can be classified into three types: Case A tensile, Case A shear, and Case B shear cracks, explained as follows.

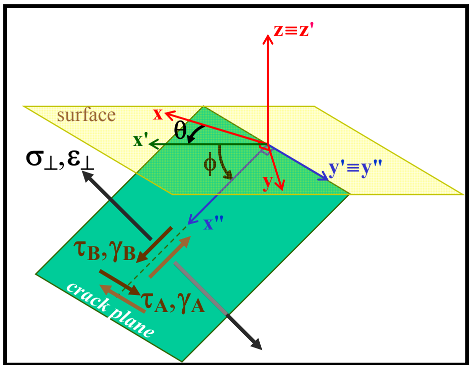

Fig. 1: Stress and strain components acting on a candidate plane (θ, ϕ) under free surface conditions, possibly with a surface pressure $\mathbf{p} = -\sigma_z \geq \mathbf{0}$.

If the surface is unpressurized ($\mathbf{p} = \mathbf{0}$), then for any given θ and any loading history it follows from Eq. (1) that both the ranges and the magnitudes of $\sigma_\perp$, τ_A, $\varepsilon_\perp$ and γ_A are maximized for planes which obey $\sin^2\phi = \sin\phi = 1$, i.e. for candidate planes $(\theta, 90^o)$ perpendicular to the free surface. This conclusion is valid even for non-proportional (NP) VAL histories, since the factors $\sin^2\phi$ or $\sin\phi$ multiply these stresses and strains for every single cycle. Fatigue cracks that initiate in planes with $\phi = 90^o$ are called Case A cracks, which can be driven either in Mode I if $\sigma_\perp$ and $\varepsilon_\perp$ dominate, or in Mode II if τ_A and γ_A dominate the damage process, generating respectively the Case A tensile and Case A shear cracks.

If a surface pressure $\mathbf{p} > \mathbf{0}$ is applied, Case A cracks would not be affected because the magnitude of the normal stress perpendicular to their planes $\sigma_\perp(\theta, \phi)$ would still be maximized for $\phi = 90^o$, hence in such cases all terms involving $\mathbf{p}$ would vanish in the above equations. Note that large pressure variations $\Delta\mathbf{p}$, due e.g. to Hertzian contact fatigue, could maximize the normal strain range $\Delta\varepsilon_\perp(\theta, \phi)$ for $\phi = 0^o$, however a $\phi = 0^o$ critical plane parallel to the free surface would be unlikely since the associated normal stress $\sigma_\perp(\theta, 0^o) = -\mathbf{p}$ would always be compressive, so it would inhibit the crack propagation process.

On the other hand, Eq. (1) shows that the ranges and magnitudes of τ_B and γ_B are maximized for planes with $\sin 2\phi = 1$, i.e. in candidate planes making 45^o with the free surface. The cracks that initiate in such $\phi = 45^o$ inclined planes are called Case B cracks, which are always driven in Mode III (due to the maximization of the τ_B and γ_B ranges and