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Stress distribution at the stiffened edges of a thick curved flat bar

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Abstract

The present paper focuses on the determination of stress distribution at the stiffened edges of a thick curved flat bar using a new computational elasticity approach. The bar is rigidly fixed at its one end and the two opposing curved edges are subjected to circumferential stiffeners. The corresponding stress field of the curved bar is obtained from the finite-difference solution of an elliptic partial differential equation of equilibrium in polar coordinate system. The analysis is carried out for two different conditions of loading at the bar end. The effect of bar aspect ratio on the state of stresses at the stiffened edges is also included in the present analysis.

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Keywords: Stress distribution; curved flat bar; circumferential stiffener; finite-difference method.

1. Introduction

The use of stiffeners in the construction of engineering structures is quite extensive. In the analysis of stiffened structures, the physical conditions of stiffeners are mathematically modeled usually in terms of a mixed mode of boundary conditions. However, the earlier mathematical models of elasticity were very deficient in handling the practical stress problems, as most of them are of the mixed-boundary-value type. Stress-analysis of structural components is still suffering from a lot of shortcomings and thus it is constantly coming up in the literature [1-4]. Among the existing mathematical models for plane elastic problems, the stress function approach and the two displacement parameter approach [5] are noticeable. Stress function approach can only take boundary conditions in terms of loadings and thus, is inadequate for practical problems. Again, the two displacement parameter approach

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involves finding two functions simultaneously satisfying two second order partial differential equations which is extremely difficult and serious attempts had hardly been made in the field of stress analysis using this formulation.

Nomenclature

r, θ	polar coordinates	ψ	potential function
E, ν	elasticity modulus and poisson's ratio respectively	h, k	mesh lengths in r- and θ-direction respectively
u_r, u_θ	radial and circumferential components of displacement respectively	i, j	nodal coordinate system in r- and θ-direction respectively
σ_r, σ_θ, $\tau_{r\theta}$	radial, circumferential and shearing components of stress respectively	r_i, r_o	inner and outer radius respectively

Prior to the widespread use of computer, basically two methods were available for stress analysis of curved beams and bars, which are Timoshenko's elasticity formulation [5] and Winkler's theory [6]. Winkler's theory mainly gives the analytical expression for the circumferential stress and the results are not valid when the curved beams are thick, because this theory neglects the effects of transverse shear deformation. The more refined Timoshenko's formulation relaxes the normality assumption of the plane sections which are to remain plane and normal to the deformed centerline of the curved beam. By allowing a further rotation of the normal, the theory admits a nonzero shearing strain. Recently, Sloboda and Honarmandi [7] have developed an elasticity based method for the analysis of curved beam of non-rectangular cross section. This method has the similar characteristic of the stress function approach, that is, it accepts boundary conditions only in terms of boundary loadings.

Stress analysis of structural problem is mainly handled by numerical methods. The major numerical methods in use are (a) the finite-difference method (FDM) and (b) the finite-element method (FEM). FDM is an ideal approach for solving partial differential equations. However, the method does not work well in case of complex boundary shapes. In that case, the FEM is the only recourse open to us. But the uncertainties associated with the FE prediction of stresses at the surface of structural components have been pointed out by several researchers [4, 8]. On the other hand, the accuracy of FDM in reproducing the state of stresses along the boundary surfaces was verified to be much higher than that of FEM analysis [4]. Moreover, solution of the present curved bar problem using FDM will not be appropriate in Cartesian environment as it invites several approximations in modeling the curved boundary. Recently Deb Nath et al. [9] have reported an analytical solution for short stiffened flat composite bar using a new elasticity based formulation, but to the author's knowledge no such investigation for short stiffened flat curved bar is available in the literature.

This paper represents a new computational investigation of the stress field in thick, stiffened curved flat bars. More specifically, the paper is on the determination of stress distributions at the inner and outer edges of the bar which are subjected to rigid circumferential stiffeners. The bar is fixed at one end and the other end is subjected to uniform shear and tensile loadings. The problem is formulated as a plane problem of cylindrical coordinate system, in terms of a potential function derived under plane stress assumption and is numerically solved using finite-difference method. The effect of bar aspect ratio on the distribution of stresses is also included in the analysis.

2. Mathematical modelling

With reference to the cylindrical coordinate system, in absence of body forces, the equilibrium equations for general isotropic materials, in terms of displacement components u_r and u_θ for plane problems of elasticity are [5]:

$$\frac{1-\nu}{2}\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1+\nu}{2}\frac{1}{r}\frac{\partial^2 u_r}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{1-\nu}{2}\frac{1}{r}\frac{\partial u_\theta}{\partial r}+\frac{3-\nu}{2}\frac{1}{r^2}\frac{\partial u_r}{\partial\theta}+\frac{\nu-1}{2}\frac{1}{r^2}u_\theta=0 \tag{1a}$$

$$\frac{\partial^2 u_r}{\partial r^2}+\frac{1+\nu}{2}\frac{1}{r}\frac{\partial^2 u_\theta}{\partial r\partial\theta}+\frac{1-\nu}{2}\frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\nu-3}{2}\frac{1}{r^2}\frac{\partial u_\theta}{\partial\theta}-\frac{u_r}{r^2}=0 \tag{1b}$$