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## Stress State of a Radial Inhomogeneous Semi Sphere under the Vertical Uniform Load

Vladimir I. Andreev<sup>a\*</sup>, Daniil A. Kapliy<sup>a</sup>

<sup>a</sup> *Moscow State University of Civil Engineering, Yaroslavl highway, 26, Moscow, 129337, Russia*

### Abstract

There is a solution of the problem of stress-strain state determining in the radially inhomogeneous concrete semi sphere. The inhomogeneity is induced by the temperature field. The problem reduces to a differential equation system with variable coefficients. The allowance for the variable Young's module lets to arrive to a more accurate solution.

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### 1. Introduction

In most constructions, which are used nowadays, elements have unchanged geometrical shape along the whole length as well as constant mechanical-and-physical properties. Stresses in such constructions are unevenly spread, the limit state can occur only in insignificant areas, the resource of the material is not completely used, which leads to its overspending. One of the promising areas of structural mechanics is to develop methods that allow you to more fully use the strength of material resource. Such methods can reduce the deformation characteristics of materials, i.e. reduce the stress that can reduce the thickness of the reinforced concrete shell, more usefully spread reinforcement in section, increase peak load. In works [1-3 and others] were developed methods to solve problems of elasticity theory for bodies with continuous inhomogeneity of deformation characteristics (elastic modulus  $E$  and Poisson's ratio  $\nu$ ), including ones which are due to temperature field. Unlike previous works, in this paper we consider the problem for the case when an inhomogeneous semi sphere stands under the vertical uniform load.

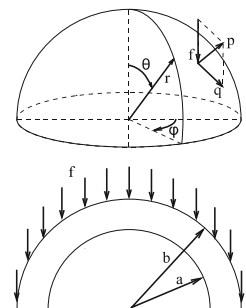


Fig. 1. The model.

\* Corresponding author. Tel.: +7-499-183-5557; fax: +7-499-183-5742.  
E-mail address: [asv@mgsu.ru](mailto:asv@mgsu.ru)

## 2. Statement of the problem

As a mechanical model we chose a thick-walled reinforced shell (semi sphere), the inner radius is  $a$ , the outer radius is  $b > a$ . The shell parameters:  $a = 3.3$  m,  $b = 4.5$  m;  $T_a = 500^\circ\text{C}$  – temperature at the inner face of the sphere;  $T_b = 0^\circ\text{C}$  – temperature at the outer face of the sphere;  $f = 1\text{MPa}$  – vertical load distributed over the entire outer face (Fig. 1).

## 3. Solution of the problem

In the case of steady state of the temperature distribution the solution of the heat equation in a spherical shell has the form:

$$T(r) = \frac{1}{b-a} \left[ (T_a - T_b) \frac{ab}{r} + T_b b - T_a a \right]. \tag{1}$$

Forced temperature deformations at a constant coefficient of linear thermal expansion coefficient equal:

$$\varepsilon_T = \alpha_T T(r).$$

In the calculations it was assumed  $\alpha_T = 0,1 \cdot 10^{-4} \text{ } ^\circ\text{C}^{-1}$

The dependence of the modulus of elasticity on temperature [3] can be approximated by the polynomial (Fig. 2):

$$E(T(r)) = E_0 \cdot \sum_{i=1}^N \beta_i T(r)^i$$

Displacement equations of equilibrium corresponding to the axisymmetric torsion-free problem:

$$\mu \nabla^2 u + 3(\lambda + \mu) \frac{\partial \varepsilon_m}{\partial r} - \frac{2\mu}{r^2} \left( u + \frac{\partial v}{\partial \theta} + v \text{ctg } \theta \right) + 3 \frac{\partial \lambda}{\partial r} \varepsilon_m + 2 \frac{\partial \mu}{\partial r} \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial \mu}{\partial \theta} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \right) - 3 \frac{\partial}{\partial r} (K \varepsilon_T) + R = 0, \tag{2}$$

$$\mu \nabla^2 v + \frac{3(\lambda + \mu)}{r} \frac{\partial \varepsilon_m}{\partial \theta} + \frac{\mu}{r^2} \left( 2 \frac{\partial u}{\partial \theta} - \frac{v}{\sin^2 \theta} \right) + \frac{3}{r} \frac{\partial \lambda}{\partial \theta} \varepsilon_m + \frac{\partial \mu}{\partial r} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \right) + \frac{2}{r^2} \frac{\partial \mu}{\partial \theta} \left( u + \frac{\partial v}{\partial \theta} \right) - \frac{3}{r} \frac{\partial}{\partial \theta} (K \varepsilon_T) + \Theta = 0, \tag{3}$$

where  $\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right)$ ;  $3\varepsilon_m = \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{2u}{r} + \frac{v}{r} \text{ctg } \theta$ .

In this problem Young's module is a function of one variable (radius). If in the axisymmetric problem in spherical coordinates  $\varepsilon_T$  is a function of one variable and the inhomogeneity is radial, then equations (2), (3) can be simplified:

$$\mu \nabla^2 u + 3(\lambda + \mu) \frac{\partial \varepsilon_m}{\partial r} - \frac{2\mu}{r^2} \left( u + \frac{\partial v}{\partial \theta} + v \cot \theta \right) + 3 \frac{\partial \lambda}{\partial r} \varepsilon_m + 2 \frac{\partial \mu}{\partial r} \frac{\partial u}{\partial r} - 3 \frac{\partial}{\partial r} (K \varepsilon_T) + R = 0, \tag{4}$$

$$\mu \nabla^2 v + \frac{3(\lambda + \mu)}{r} \frac{\partial \varepsilon_m}{\partial \theta} + \frac{\mu}{r^2} \left( 2 \frac{\partial u}{\partial \theta} - \frac{v}{\sin^2 \theta} \right) + \frac{\partial \mu}{\partial r} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \right) + \Theta = 0, \tag{5}$$

The stress boundary conditions for the axisymmetric problem can be formulated in the following way:

$$r = a, \sigma_r = -p_a, \tau_{r\theta} = -q_a; \quad r = b, \sigma_r = -p_b, \tau_{r\theta} = q_b. \tag{6}$$

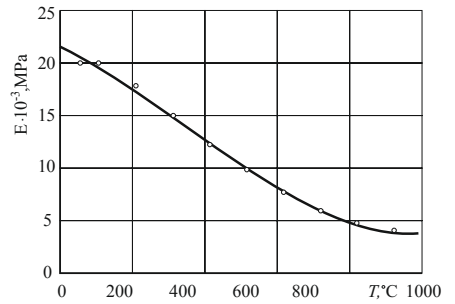


Fig. 2. Experimental data (◦) of the Young's modulus depending on the temperature of concrete and the graph of the approximating function  $E(T)$ .

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