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# Non-Stationary Flexural Fluctuations of a Round Flat Bimorph Plate with Graded -Varying Thickness

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**Abstract**

The subject matter of the article is a non-stationary axially symmetric goal for a round flat bimorph plate affecting end faces of electric potential which in itself is an arbitrary time function. The designed model allows to take into account the peculiarity of the electric-field pattern based on same inner electrode, which appears to be a metal support plate. The authors apply the method of finite integral transformation (based on Timoshenko's theory) to develop a new closed solution for the electro-elastic system of graded-varying stiffness and thickness under consideration. The achieved ratio design allows to further analyze frequency and stress-strain behaviour of resonant flexural piezo-ceramic converters.


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*Keywords:* the Goal of Inverse Piezo Effect; the Flat Bimorph Plate; the Axially Symmetric Dynamic Load.

## 1. Introduction

In various acoustic resonant devices thin bi-morph plates are used as energy converters. These systems have high electromechanical parameters and usually consist of round metallic substrate with piezo-ceramic plates of a smaller diameter attached on both sides. Bending vibrations are carried out by summing piezo-elements with electric potential to the end surfaces with electrodes. Calculation of the bodies of rotation is performed by fitting a set of ring and solid plates of constant thickness. In this study of the stress-strain state of each element, the technical theory of thin plates [1, 2] is used, which can be applied only at the low-frequency external influence. However, in modern ultrasonic resonant converters operating frequency is up to 100 kHz.

In this paper we study a thin bi-morph piezo-ceramic plate on the basis of the Timoshenko hyperbolic system, which greatly enhances the study of high-dynamic wave processes [3]. In addition, based on equation of electrostatics, in contrast to [4], tailored distribution of the electric field in the piezoelectric ceramic plates separated by an internal electrode of a metal substrate is taken into account.

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## 2. Problem considerations

Let a bi-morph circular plate consist of a metal substrate, which in the cylindrical coordinate system $(r_*, \theta, z)$ occupies region $\Omega$ : $\{0 \le r_* \le b, 0 \le \theta \le 2\pi, -h_1^*/2 \le z \le h_1^*/2\}$ and two piezo-ceramic elements $\{0 \le r_* \le a, a < b\}$ glued on it with thickness $h_2^*$ (Fig. 1). Axisymmetric bending oscillations are excited by summing the electrodes, which are located on the piezo-plates' end surfaces, with electric voltage $V^*(t_*)$ which is an arbitrary function of time $t_*$. Conditions of fixing the cylindrical surfaced plate may be arbitrary. We assume that it is fixed rigidly.

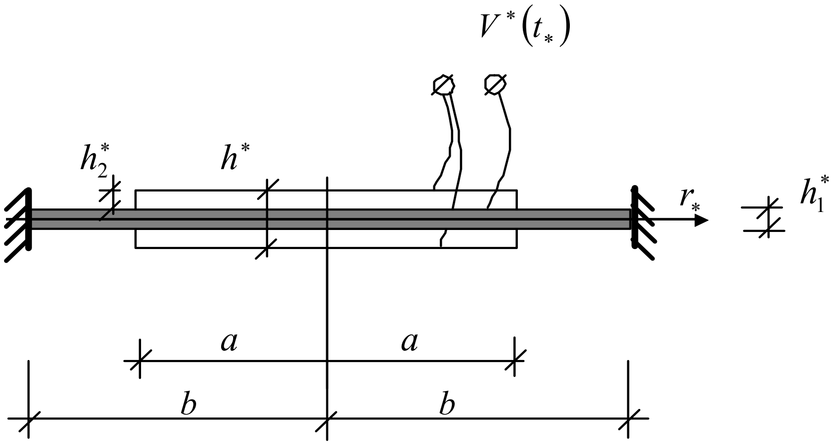


Fig. 1. Bi-morph plate

The equations of translational and rotational motion for thin plates relative to the transverse force $Q_r$ and bending moments $M_r$, $M_\theta$ are the following [5]

$$\frac{\partial Q_r}{\partial r_*} + \frac{Q_r}{r_*} - \rho(r_*) h^*(r_*) \frac{\partial^2 W^*}{\partial t_*^2} = 0$$
$$\frac{\partial M_r}{\partial r_*} - Q_r + \frac{M_r - M_\theta}{r_*} + \rho(r_*) \frac{h^*(r_*)^3}{12} \frac{\partial^2 \psi}{\partial t_*^2} = 0 \tag{1}$$

where $W^*(r_*, t_*)$ $\psi(r_*, t_*)$ deflection and rotation angle of the cross section in the $(r_*, z)$ plane, respectively, while $\rho(r)$ and $h^*(r_*)$ are density and thickness of the bi-morph plate, respectively.

Based on the equation of electrostatics [6], normal component of tension in the piezo-ceramic plates of the a composite structure is defined by equation [7]

$$E_z(r_*, z, t_*) = \left[ \frac{V^*(t_*)}{h_2^*} f(z) - \frac{e_{31}}{\varepsilon_{33}} \left( \frac{\partial \psi}{\partial r} + \frac{\psi}{r} \right) \left( z - \frac{h^*}{4} f(z) \right) \right] H(a - r_*). \tag{2}$$

Expressions for the bending moments and shear forces are determined, taking into account (2), by the following equalities [8]