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Statistically Speaking

Percent Differences: Another Look

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Introduction

This article discusses 3 common measures of comparison (absolute difference, ratio, and percent difference [PD]), explores the advantages and disadvantages of each, introduces a less-common measure called symmetrized percent change, and explains how it overcomes some of the disadvantages of the other measures.

Background

Researchers often need to compare 2 quantitative measures and summarize the difference succinctly. The comparison can be within individuals, such as diastolic blood pressure before and after an intervention, or it can be between groups, such as the average heart rate in men compared with that in women. Surprisingly, there is no standardized way to summarize these comparisons.

One common-sense option is the absolute difference (D): $D = A - B$, where A and B are the 2 quantitative measures to be compared. Another option is the ratio of A to B: $R = \frac{A}{B}$. A variant of the ratio is the PD of A relative to B: $PD = \frac{A-B}{B} \times 100 = (R - 1) \times 100$. See Box A for an example.

The absolute difference has the advantage of being in the units of the original measures, which makes it easy to interpret. It is also symmetric between measures and is not affected by reversing the direction of comparison, that is, if the difference in individual's weight after an

Box A. Three measures of comparison, after a change in units and reversal of direction

Example:

Total serum cholesterol of an individual at Time A = 200 mg/dL

Total serum cholesterol of an individual at Time B = 210 mg/dL

Comparison	Difference (D)	Ratio (R)	PD
B vs A, original units	210 mg/dL − 200 mg/dL = +10 mg/dL	210/200 = 1.05	100*(210 − 200)/200 = 100*(1.05 − 1) = +5%
Changing the units: B vs A, new units	5.4306 mmol/L − 5.172 mmol/L = +0.2586 mmol/L	5.4306/5.172 = 1.05	100*(5.4036 − 5.172)/5.172 = 100*(1.05 − 1) = +5%
Reversing the direction: A vs B, original units	200 − 210 = −10 mg/dL	200/210 = 0.9524	100*(200 − 210)/210 = 100*(0.9524 − 1) = −4.76%
Symmetry	Symmetric	Not symmetric	Not symmetric
Dependence on units of measurement	Dependent on units	Independent of units	Independent of units
Ease of interpretation	Easier to interpret	Harder to interpret	Easier to interpret

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Box B. Growing asymmetry in PDs for large versus small changes

Example:	Study 1: Smaller Difference			Study 2: Larger Difference		
	Group A mean systolic BP: 150 mm Hg Group B mean systolic BP: 120 mm Hg			Group A mean systolic BP: 150 mm Hg Group B mean systolic BP: 80 mm Hg		
Group B vs Group A	D = −30 mm Hg	R = 0.8	PD = −20%	D = −70 mm Hg	R = 0.53	PD = −46.7%
Group A vs Group B	D = +30 mm Hg	R = 1.25	PD = +25%	D = +70 mm Hg	R = 1.875	PD = +87.5%

intervention is −20 lbs compared with weight before an intervention, then the weight before is +20 lbs compared with after. A comparison of Row 3 and Row 1 in the table in Box A show this symmetry.

A benefit of the ratio and the PD is that they are measures of relative change and therefore are independent of unit of measure, so that differences can be compared across situations with different measures. A comparison of Row 2 and Row 1 in the table in Box A show this independence.

Absolute differences and PDs are also relatively easy to interpret, whereas ratios often need to be converted into the PD form to be understandable in plain language. There are significant disadvantages to ratios and PDs, which are explained in more detail in the next section.

Potential Pitfalls of Measures of Relative Change

Unlike absolute differences, ratios and PDs are not symmetric across measures [1]. For example, as seen in Rows 3 and 1 in Box A, the total serum cholesterol level at Time B is 5% higher than the cholesterol level at Time A, but the reverse is not true: the cholesterol level at Time A is only 4.76% lower than that at Time B.

This potentially confusing asymmetry means that the choice of baseline can be very important. There are often natural baselines, such as the earliest time point or a control group. However, as Cole and Altman [1] point out, this is often not the case: they give an example in which, among British 20-year olds, men are 8.4% taller than women, but women are 7.7% shorter than men. Both measures are correct.

There is a certain cognitive dissonance in this asymmetry, which makes PDs tricky to interpret. The problem stems from our desire to have a unit-free, relative measure of difference, because we are always left with the question: difference relative to what? A penny given to a pauper is of much greater relative importance than a penny given to the wealthy; simple PD tries to capture this importance by forcing one to choose a baseline to measure the difference against. If the average woman were to grow to an average man's height, she would gain 8.4 cm for every 100 cm she originally had (because she had fewer centimeters to begin with), whereas the same man shrinking to a woman's height would lose only 7.7 cm for his every 100 cm.

Accordingly, when the difference between 2 measures is small, the PD is nearly symmetric, as was seen in Box A with PDs of 5% and 4.76%. But when 2 measures have bigger differences, the asymmetry is more pronounced, as illustrated in Box B. With an absolute difference in systolic BP of 30 mm Hg, mean systolic BP in Group B was 20% lower than in Group B, and 25% higher in Group A than in Group B. In a second study, the absolute difference was 70 mm Hg, and now the discrepancy is larger: mean systolic BP was 46.7% lower in Group B than in Group A but 87.5% higher in Group A than in Group B.

PDs are also sensitive to the size of the baseline. Box C shows how the same absolute difference can lead to greatly increased PDs as the baseline value shrinks. A drop in the number of lesions in an individual from 110 to 100 yields a 10% difference, whereas a drop from 11 to 1 yields a 1000% difference.

Box C. Sensitivity of PDs to baseline size

Comparison of number of lesions within 1 individual over 2 time points

Comparison Values	Absolute Difference (D)	PD
110 vs 100	10	10%
60 vs 50	10	20%
15 vs 5	10	200%
11 vs 1	10	1000%
10 vs 0	10	Infinity/ undefined