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Original Article

Validation of a Mathematical Model of Bidirectional Glenn Circulation With Aortopulmonary Collaterals and the Implications for Q_P/Q_S Calculation

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Objectives: A mathematical model of the oxygen delivery kinetics of the bidirectional Glenn (BDG) shunt circulation incorporating aortopulmonary collateral (APC) flow was created. The model was used to characterize oxygen delivery and compare modeled data to actual patient data obtained using cardiac magnetic resonance imaging (MRI) and catheterization. In addition, cardiac MRI and catheterization assessment of pulmonary blood flow in the presence of APC flow were compared.
Design: Mathematical model and retrospective data analysis of patients who underwent cardiac MRI and catheterization. The mathematical model is based on the concept that APC flow to the lungs is recirculated pulmonary venous blood flow, which does not contribute to systemic oxygen delivery.
Setting: Single-center, university teaching hospital.
Participants: The study included 98 patients with BDG shunt undergoing cardiac MRI and cardiac catheterization.
Measurements and Main Results: In the absence of APC flow, the pulmonary blood flow to systemic blood flow ratio (Qp/Qs) calculated using cardiac catheterization data closely matched that obtained with cardiac MRI. In the presence of APC flow, Qp/Qs calculated using cardiac catheterization data systematically underestimated values obtained with cardiac MRI. A mathematical model of BDG shunt oxygen delivery incorporating variable APC flow was created. The model provided reasonable prediction of actual patient data for arterial blood oxygen, superior vena cava oxygen saturation, and oxygen delivery obtained at the time of cardiac catheterization in patients.
Conclusion: The oxygen delivery kinetics of a BDG shunt incorporating variable APC flow can be modeled mathematically. Model output can be used to predict blood oxygen saturation after coil embolization of APC flow in the cardiac catheterization laboratory.

Key Words: mathematical model; single ventricle; bidirectional Glenn; aortopulmonary collateral; cardiac magnetic resonance imaging; cardiac catheterization; Qp/Qs

PREVIOUSLY, THE AUTHORS AND OTHERS have mathematically modeled the oxygen delivery kinetics of the superior cavopulmonary anastomosis (bidirectional Glenn [BDG] shunt) physiology, which is used as a transitional circulation in patients with single ventricle physiology before final palliation with the Fontan procedure.[1,2] Many of these BDG patients develop aortopulmonary collateral vessels (APCs)[3–5] that provide an additional source of pulmonary blood flow. None of the previously published models

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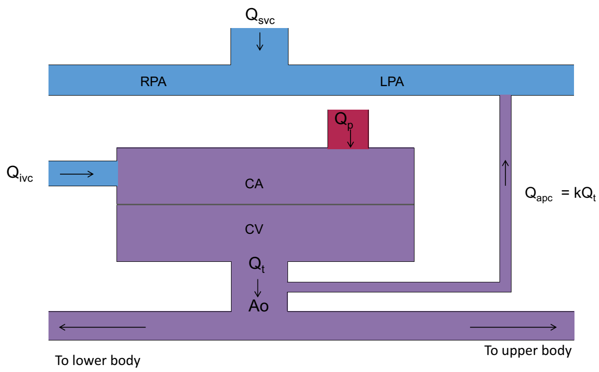

Fig 1. Bidirectional Glenn shunt with aortopulmonary collateral. Ao, aorta; CA, common atrium; CV, common ventricle; LPA, left pulmonary artery; RPA, right pulmonary artery.

incorporates variables to account for this flow (Fig 1). In this investigation using BDG patients undergoing cardiac magnetic resonance imaging (MRI) and cardiac catheterization as part of a pre-Fontan evaluation, the authors sought to (1) develop a mathematical model of BDG oxygen delivery kinetics incorporating APC flow, (2) compare Qp/Qs determined using cardiac catheterization to that determined using cardiac MRI, and (3) evaluate the performance of their model using MRI-determined pulmonary blood flow to systemic blood flow ratio (Qp/Qs) in predicting data obtained at cardiac catheterization.

Methods

Mathematical Model of BDG Shunt With APCs

The mathematical model of BDG shunt with APCs is described as follows. The abbreviations used for the equations throughout the article are defined in Table 1.

In the steady state, total body oxygen consumption equals lung oxygen uptake (SVO_2 = whole body oxygen consumption [CVO_2]).

If p is the fraction of CVO_2 consumed by the upper body and (1-p) is the fraction consumed by the lower body, then

$$CVO_2 = p \times CVO_2 + (1-p) \times CVO_2 \quad (1)$$

$$C_{IVC}O_2 \times Q_{IVC} = CaO_2 \times Q_{IVC} - (1-p) \times CVO_2 \quad (2)$$

$$C_{SVC}O_2 \times Q_{SVC} = CaO_2 \times Q_{SVC} - p \times CVO_2 \quad (3)$$

If upper and lower body oxygen consumption is coupled to superior vena cava (SVC) and inferior vena cava (IVC) flow respectively,

$$Q_{SVC} = p \times (Q_T - Q_{APC}) \quad (4)$$

$$Q_{IVC} = (1-p) \times (Q_T - Q_{APC}) \quad (5)$$

k is defined as the ratio of APC blood flow/ventricular cardiac output,

$$Q_{APC} = kQ_T \quad (6)$$

Table 1
Abbreviations Used in Mathematical Model

CaO_2	Arterial oxygen content
$C_{IVC}O_2$	Inferior vena cava oxygen content
$CpvO_2$	Pulmonary vein oxygen content
$C_{SVC}O_2$	Superior vena cava oxygen content
CVO_2	Whole body oxygen consumption
DO_2	Systemic oxygen delivery
PaO_2	Arterial partial pressure of oxygen
Q_{APC}	Aortopulmonary collateral blood flow
Q_{AscAo}	Ascending aortic blood flow
Q_{DscAo}	Descending aortic blood flow
Q_{IVC}	Inferior vena cava blood flow
Q_{LLPV}	Left lower pulmonary vein blood flow
Q_{LPA}	Left pulmonary arterial blood flow
Q_{LUPV}	Left upper pulmonary vein blood flow
Qp	Pulmonary blood flow
Q_{RLPV}	Right lower pulmonary vein blood flow
Q_{RPA}	Right pulmonary arterial blood flow
Q_{RUPV}	Right upper pulmonary vein blood flow
Qs	Systemic blood flow
Q_{SVC}	Superior vena cava blood flow
Q_T	Total ventricular output
SaO_2	Arterial blood oxygen
$S_{IVC}O_2$	Inferior vena cava oxygen saturation
$S_{SVC}O_2$	Superior vena cava oxygen saturation
$S_{PA}O_2$	Pulmonary arterial oxygen saturation
$SpvO_2$	Pulmonary vein oxygen saturation
SpO_2	Blood oxygen saturation
SVO_2	Systemic oxygen delivery

$$Qs = Q_T - Q_{APC} \quad (7)$$

$$DO_2 = CaO_2 \times (Q_T - Q_{APC}) = CaO_2 \times (Q_T - kQ_T) = (1-k) \times CaO_2 \times Q_T \quad (8)$$

In BDG physiology, oxygen delivery from the common ventricle is the sum of oxygen delivery to the common atrium,

$$CaO_2 \times Q_T = CpvO_2 \times Qp + C_{IVC}O_2 \times Q_{IVC} \quad (9)$$

Using formula (2), (9) is rewritten as

$$CaO_2 \times Q_T = CpvO_2 \times Qp + CaO_2 \times Q_{IVC} - (1-p) \times CVO_2 \quad (10)$$

From (5) and (6),

$$Q_{IVC} = (1-p)(Q_T - kQ_T) = (1-p)(1-k) \times \cdot Q_T \quad (11)$$

From (10) and (11),

$$CaO_2 \times Q_T = CpvO_2 \times Qp + (1-p)(1-k) \times CaO_2 \times Q_T - (1-p) \times CVO_2,$$

which simplifies to,

$$(k + p - kp)CaO_2 \times Q_T = CpvO_2 \times Qp - (1-p) \times CVO_2 \quad (12)$$

$$Qp = Q_{SVC} + Q_{APC} \quad (13)$$

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