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Limiting phase trajectories as an alternative to nonlinear normal modes

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Abstract

We discuss a recently developed concept of limiting phase trajectories (LPTs) allowing a unified description of resonance, highly non-stationary processes for a wide range of classical and quantum dynamical systems with constant and varying parameters. This concept provides a far going extension and adequate mathematical description of the well-known linear beating phenomenon to a diverse variety of nonlinear systems ranging from classical multi-particle models to nonlinear quantum tunneling. While stationary (and non-stationary, but non-resonant) oscllations can be described in the framework of non-linear normal modes (NNMs) concept, it is not so in the considered case of resonant non-stationary processes. In the latter case which is characterized by intense energy exchange between different parts of a system, an additiional slow time scale appears. The energy exchange proceeds in this time scale and can be identified as strong modulation of the fast oscillations. The aforementioned resonant non-staionary prcesses include, e.g., targeted energy transfer, non-stationary vibrations of carbon nanotubes, quantum tunneling, auto-resonance and non-conventional synchronization. Besides the non-linear beating, the LPT concept allows one to find the conditions of transition from intense energy exchange to strongly localized (e.g. breather-like) excitations. A special mathematical technique based on the non-smooth temporal transformations leads to the clear and simple description of strongly modulated regimes. The role of LPTs in the theory of resonance non-stationary processes turns out to be similar to that of NNMs in stationary case.

As an example we present results of analytical and numerical study of planar dynamics of a string with uniformly distributed discrete masses without a preliminary stretching. Each mass is also affected by grounding support with cubic characteristic (which is equivalent to transversal unstretched string). We consider the most important case of low-energy transversal dynamics. This example is especially instructive because the considered system cannot be linearized. Adequate analytical description of resonance non-stationary processes which correspond to intensive energy exchange between different parts of the system (clusters) in low frequency domain was obtained in terms of LPTs. We have revealed also in these terms the conditions of energy localization on the initially excited cluster. Analytical results are in agreement with the results of numerical simulations. It is shown that the considered system can be used as an efficient energy sink.

Keywords: Weakly Coupled Nonlinear Oscillators; Nonlinear Normal Mode; Energy Transfer; Energy Localization; Limiting Phase Trajectory; un-stretched string; acoustic vacuum; energy sink; Poincaré sections; transition to chaotic dynamics;

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1. Introduction

The accepted classification of the problems of mathematical physics (in application to models of the oscillation and wave theory) draws first of all a sharp distinction between linear and nonlinear model[1,2,3,4,5]. Such a distinction is caused by understandable mathematical reasons including absence of superposition principle in the nonlinear case. However it was recently shown[6,7,8,9,10,11,12,13] that in-depth physical analysis allows us to introduce other basis for classification of the oscillation problems, focusing on the difference between stationary (or non-stationary, but non-resonance) and resonance non-stationary processes. In the latter case a discrimination of linear and nonlinear problems is not fundamental if we deal with regular (non-chaotic) motions, and a specific technique has been developed which is efficient in the same degree for description of both linear and nonlinear resonance non-stationary processes. The existence of two alternative approaches in the framework of linear theory seems unexpected. Really, the superposition principle allows us to find a solution describing arbitrary non-stationary oscillations as a combination of linear normal modes which correspond to stationary processes. However, in the systems of weakly coupled oscillators, in which resonance non-stationary vibrations can occur, other type of fundamental solution exists. It describes strongly modulated non-stationary oscillations characterized by the maximum possible energy exchange between the oscillators or the clusters of the oscillators (effective particles). Such solutions are referred to as Limiting Phase Trajectories (LPTs). It was demonstrated that the LPT concept suggests a unified approach to the study of highly non-stationary processes in a wide range of classical and quantum dynamical systems with constant and time-varying parameters[12]. The development and use of analytical tool based on the LPT concept is motivated by the fact that resonance non-stationary processes occurring in a broad variety of finite dimensional physical models are beyond the well-known paradigm of nonlinear normal modes (NNMs), fully justified only for quasi-stationary and non-stationary, but non-resonance processes. While the NNMs approach has been proved to be an effective tool for the analysis of stationary regimes, their instability and bifurcations (see, e.g.,[2,3]), the use of the LPTs concept provides the adequate procedures for studying strongly non-stationary regimes as well as the transitions between different types of non-stationary motions, including propagation of localized excitations[7,8]. It makes possible, at the first time, to extend the notion of beating phenomenon to the systems with many degrees of freedom. Moreover, the concept of the limiting phase trajectories allows the prediction of the new type of synchronization (LPT-synchronization) in the system of weakly coupled autogenerators[9] and this is in contrast to the conventional NNM-synchronization[14]. Note that, along with the well-known asymptotic methods, the investigation of the phenomena under discussion has required the application of the special technique of non-smooth temporal transformations providing a simple description of strongly modulated and transient regimes. This technique was initially elaborated for description of vibro-impact (or close to them) processes[15].

In this paper we demonstrate the role of the LPT concept in Non-stationary Resonance Dynamics and its relation to the NNM concept on the example of unstretched string with grounding cubic supports undergoing predominant transversal motion. It was shown recently[16] that in the limit of low energy a fixed-fixed chain of linearly coupled particles performing in-plane transverse oscillations possesses strongly nonlinear dynamics and acoustics due to geometric nonlinearity, forming a nonlinear acoustic vacuum. This designation denotes the fact that the speed of sound as defined in the sense of classical acoustic theory is zero in that medium, so the resulting equations of motion lack any linear stiffness components. A significant feature of that system was the presence of strongly non-local terms in the governing equations of motion (in the sense that each equation directly involves all particle displacements), in-spite of the fact that the physical spring-mass chain has only local (nearest-neighbor) interactions between particles. These non-local terms constitute a time-dependent 'effective speed of sound' for this medium, which is completely tunable with energy. A rich structure of resonance manifolds of varying dimensions were identified in the nonlinear sonic vacuum, and 1:1 resonance interactions are studied asymptotically to prove the possibility of strong energy exchanges between nonlinear modes.

One of the distinctive features of a chain without grounding support was that its nonlinear normal modes – NNMs[3] could be exactly determined. Moreover, the analysis has shown that the number of NNMs in the sonic vacuum is equal to the dimensionality of the configuration space and that no NNM bifurcations are possible. In addition, the most intensive 1:1 resonance intermodal interaction was the one realized by the two NNMs with the highest wave numbers. However, the unstretched string model considered in[16] is in some sense a special case, since one of the most significant features of dynamical systems with homogeneous potentials is that the number of NNMs may exceed the number of degrees of freedom due to mode bifurcations[1]. One can expect that such NNM bifurcations will also lead