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On the Transverse Vibrations of Strings and Beams on Semi-Infinite Domains

Tugce Akkaya[a,*], Wim T. van Horssen[a]

[a] *Delft Institute of Applied Mathematics*
Delft University of Technology
Mekelweg 4, 2628 CD Delft, The Netherlands

Abstract

In this paper, we study the transverse vibrations of a string and of a beam which are infinitely long in one direction. These vibration problems can be used as a toy model for rain-wind induced oscillations of cables. In order to suppress undesired vibrations in the string (or beam), dampers are used at the boundary. The main aim of this paper is to show how solutions for these string and beam problems on a semi-infinite domain can be computed. We derive explicit solutions for a linear string problem which is attached to a mass-spring-dashpot system at $x = 0$ by using the D'Alembert method, and for a transversally vibrating beam problem which has a pinned, sliding, clamped or damping boundary, respectively, at $x = 0$ by using the method of Laplace transforms. It will be shown how waves are reflected for different types of boundaries.

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Keywords: Boundary damper ; strings ; beams ; D'Alembert Methods ; The method of Laplace transforms

1. Introduction

In recent decades, among both applied mathematicians and engineers, research in the field of the vibrations of the stay cables of cable stayed bridges has received a lot of attention. Usually inclined stay cables of bridges are attached to a pylon at one end and to the bridge deck at the other end. Due to low structural damping of the bridge, a wind-field containing raindrops may excite a galloping type of vibrations. For example, one can refer to the Erasmus bridge in Rotterdam, which started to swing under mild wind conditions, shortly after it was opened to the traffic in 1996. To suppress the undesired oscillations of the bridge, dampers were installed, as can be seen in Figure 1. As has been observed from engineering wind-tunnel experiments, raindrops hitting the inclined stay cable cause the generation of one or more rivulets on the surface of the cable. The presence of flowing water on the cable changes the mass of the bridge system that can lead to instabilities, which are not fully understood. The vibrations of the bridge cables can be described mathematically by string-like or beam-like problems. Models for such cables can be found in [1,2]. In order to stabilize the problem, boundary damping is taken into account. The aim of this paper is to provide an understanding

* Corresponding author. Tel.: +31 (0)15 27 86435 .
E-mail address: T.Akkaya@tudelft.nl

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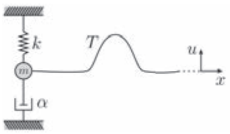
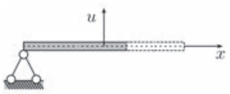
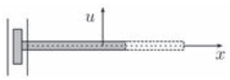
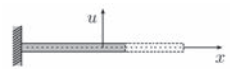
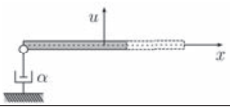
Fig. 1: Used new dampers to the Erasmus bridge to prevent vibrations. Photo: courtesy of TU Delft.

of how effective boundary damping is for string and beam equations. The reflection and damping properties of waves propagating at a non-classical boundary for wave equations on a semi-infinite interval was studied in[3] by using the D'Alembert formula.

Table 1: Boundary condition(s) for a semi-infinite string (Model 1) and a beam (Model 2).

Type of system	Left end condition	Boundary conditions at $x = 0$
Model 1		
Mass-spring-dashpot[3]		$mu_{tt} = Tu_x - ku - \alpha u_t$
Model 2		
Pinned		$u = 0, \quad u_{xx} = 0$
Sliding		$u_x = 0, \quad u_{xxx} = 0$
Clamped		$u = 0, \quad u_x = 0$
Damper		$u_{xx} = 0, EIu_{xxx} = \alpha u_t$

2. The Transverse Vibrations of The String-problem (Model 1)

In this section, we will consider the perfectly flexible string of infinite extension in the positive x-direction. It is assumed that gravity and other external forces can be neglected. The vertical transversal displacement $u(x, t)$ along a string, where x is the position along the string and t is the time, satisfies the following differential equation which can be obtained by using Hamilton's principle[4]:

$$u_{tt} - c^2 u_{xx} = 0, \quad 0 < x < \infty, \quad t > 0, \tag{1}$$

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad 0 \leq x < \infty, \tag{2}$$