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The motion of a railway wheelset on a track or on a roller rig

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Abstract

The kinematic motion of a wheelset of a railway vehicle rolling on a pair of rails without slips is studied. The exact linearized motions for a wheelset on a tangent track and for a wheelset on a roller rig are derived. Furthermore, equations for the symmetric case for finite amplitudes are presented, which show that the wavelength of the kinematic motion depends on the amplitude. From a dynamic analysis, it is shown how the critical speed of a real wheelset with tyre slips and suspension stiffness relates to the kinematic motion.

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Keywords: Railway wheelset; kinematic motion; stability; critical speed

1. Introduction

The motion of a single wheelset of a railway vehicle rolling on a track or on a roller rig is considered. The wheelset consists of two wheels rigidly connected by an axle; the wheelset as well as the pair of rails are assumed to be rigid. The wheelset typically has two points of contact, one at either rail.

The configuration of a rigid body rolling and sliding on a surface with two points of contact is constrained by the two contact conditions and forms a space with four dimensions, two fewer than the configuration space of a rigid body moving in space has. If sliding at the contact points is allowed, the system has four degrees of freedom, but if the body rolls without sliding, four velocity constraints are added. Of these four constraints, only three are independent, because the velocities of the rigid body at the contact points projected on the line through the contact points are the same. An instantaneous rotation about this line is possible, which can be extended to a finite motion if some smoothness and curvature conditions are met at the contact points, which may be limited by a third point of the rigid body coming in contact with the surface. Therefore, the system has one degree of freedom and to describe the state of the system, four configuration coordinates and one velocity coordinate are needed.

The wheel profiles at both wheels are axially symmetric with the same axis of symmetry, but the profiles may differ. The profile of the rails is constant along the track, but the left- and right-hand rail may differ. The cases of a tangent track, with level prismatic rails, and a roller rig, in which the rails form hoops in a vertical plane that can rotate about their common horizontal axis, are considered. In both cases, there is a central motion in which the yaw angle is zero

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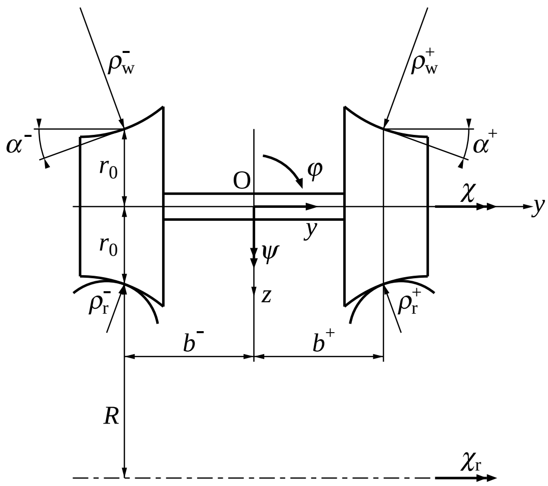

Fig. 1. Wheelset on a roller rig or on a tangent track if R is very large.

and the lateral displacement is constant, whereas the forward displacement on a tangent track, or the rotation angle of the drum with the rails of a roller rig, and the pitch angle increase linearly in time. As a simplification, it is assumed that the axis of the wheelset is horizontal for the central motion.

In the kinematic motion with the conditions of no sliding imposed, a perturbation of the central motion leads either to a growing solution until the assumptions for the two-point contact to exist are no longer fulfilled, or to a periodic motion. The periodicity comes from the symmetry of the problem under time reversal. The periodic motion for small amplitudes was first analysed by Klingel[1] and is therefore called a Klingel motion. The exact wavelength for a wheelset on a left–right symmetric tangent track was derived by the author[2] and later by Antali and Stepan[3]. Here, the wavelengths for the general asymmetric case of a tangent track and for a wheelset on a roller rig are derived. By a perturbation analysis, the small-amplitude solution can be extended for finite amplitudes.

If tangential forces are transmitted at the contact points, small slips occur, which can make the central motion unstable. For increasing speeds, the central motion changes from being stable to being unstable at a critical speed by a Hopf bifurcation. The relation of this instability to the kinematic motion is investigated.

2. Kinematic motion

2.1. General non-linear relations

A wheelset on a roller rig is shown in Fig. 1. In the central position, the radii of the wheels at the contact points are r_0 and the radii of the rails on the drum are R. The contact point at the left-hand wheel is at a distance b^- from the centre of the wheelset and the right-hand contact point is at a distance b^+ from this centre. The six coordinates that describe the configuration of the wheelset are the three displacements of the centre of the wheelset from the central position, x in the forward direction, y in the lateral direction, positive to the right, and z in the downward direction. The orientation is described by three modified Euler angles in the XZY order, that is, a roll angle φ about the initial x-axis, a yaw angle ψ about the rotated z-axis and a pitch angle χ about the axis of the wheelset in the final position. While the roll and yaw angles are generally small, the pitch angle can be arbitrarily large.

The profile of either wheel is described by specifying the wheel radius as a function of the outward distance along the centre line of the wheelset, starting from the contact point in the central position, so for the left-hand wheel, the wheel profile is described by a function $r_w^-(y_w^-)$ and for the right-hand wheel, the profile is $r_w^+(y_w^+)$. Because of the choice of the origin, we have $r_w^-(0) = r_w^+(0) = r_0$. Similarly, the profile of either rail is specified by the local z-coordinate as a function of the outward distance at the rail, both measured from the contact point in the central