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Investigation of the global instability of the rotating-disk boundary layer

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Abstract

The development of the flow over a rotating disk is investigated by direct numerical simulations using both the linearized and fully nonlinear incompressible Navier–Stokes equations. These simulations allow investigation of the transition to turbulence of the realistic spatially-developing boundary layer. The current research aims to elucidate further the global linear stability properties of the flow, and relate these to local analysis and discussions in literature. An investigation of the nonlinear upstream (inward) influence is conducted by simulating a small azimuthal section of the disk (1/68). The simulations are initially perturbed by an impulse disturbance where, after the initial transient behaviour, both the linear and nonlinear simulations show a temporally growing upstream mode. This upstream global mode originates in the linear case close to the end of the domain, excited by an absolute instability at this downstream position. In the nonlinear case, it instead originates where the linear region ends and nonlinear harmonics enter the flow field, also where an absolute instability can be found. This upstream global mode can be shown to match a theoretical mode from local linear theory involved in the absolute instability at either the end of the domain (linear case) or where nonlinear harmonics enter the field (nonlinear case). The linear simulation grows continuously in time whereas the nonlinear simulation saturates and the transition to turbulence moves slowly upstream towards smaller radial positions asymptotically approaching a global upstream mode with zero temporal growth rate, which is estimated at a nondimensional radius of 582.

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Keywords: rotating disk, boundary layer flow, direct numerical simulation

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1. Introduction

The incompressible boundary layer over a rotating disk without any imposed external flow is studied. The laminar profiles arising over the disk are shown in Fig. 1 and constitute the similarity solution of the cylindrical Navier–Stokes

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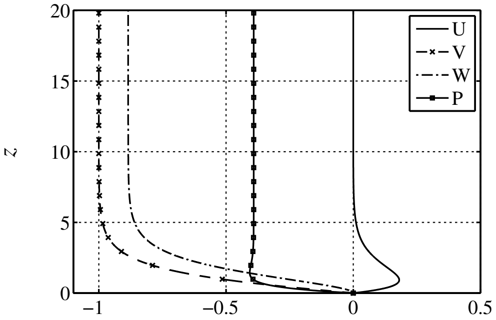

Fig. 1. The laminar velocity profiles (in the rotating reference frame) of the similarity solution for the flow over an infinite rotating disk. U is the radial velocity component, V is the azimuthal velocity component and W is the vertical velocity component. For completeness also the pressure P is included which is normalized to zero at the wall.

equations for a disk of infinite radius.[1] The boundary layer consists of a three-dimensional axisymmetric flow, with a constant thickness and a Reynolds number (R) increasing linearly with radius (r) defined by

$$R = \frac{r^*\Omega^* L^*}{\nu} = \frac{r^*}{L^*} = r, \tag{1}$$

where ν is the kinematic viscosity of the fluid, Ω^* is the rotation velocity and the nondimensionalizing lengthscale is $L^* = (\nu/\Omega^*)^{1/2}$. The star superscript refers to dimensional units where needed. The radial velocity profile ($U = U^*/(r^*\Omega^*)$) is inflectional making the flow susceptible to an inviscid crossflow instability. The stability properties have long been examined both through experiments and theory,[2,3] yet there is no clear picture of either the instability in a global frame or the precise transition mechanism eventually leading to turbulent flow.

In 1995 a local absolute instability was found to exist at a Reynolds number $R = 507$ based on theory.[3] This absolute instability appears in the local framework since it was defined by a linear stability analysis of the mean profiles at locally prescribed Reynolds numbers, ignoring the change in Reynolds number with radius. Nevertheless, this critical Reynolds number was shown to agree well with experimentally observed Reynolds numbers for the onset of nonlinearity and the subsequent transition process.[4] Thus, the discovery of the theoretical critical Reynolds number for the onset of absolute instability triggered further theoretical, experimental and numerical research.[5-8] The current research is directed towards understanding how this local concept of the absolute instability is translated to the real flow, as appearing in (real or numerical) experiments. Using experiments and model problems, both the linear and nonlinear behaviour affected by the position of the edge of the disk have been investigated.[7-9] Here, simulations including a finite extent of the linear domain are conducted, either by modelling the effect of outward turbulence (linear simulation) or with actual developed turbulence (nonlinear simulation), showing how the absolute instability behaves in its presence. The nonlinear simulation presented is used to investigate the linear behaviour in the presence of an annulus of turbulence at high R. Both simulations are part of previous work where the linear global flow was investigated.[10] For that work, the nonlinear simulation was used as a means to verify that the outer radial boundary condition for the linear domain was correctly modelling the influence of the turbulence. Here, the nonlinear simulation is described in detail and also the development in time is followed. The linear simulation is instead used as a tool to show that the behaviour uncovered remains linear. The outline is as follows. In section 2 the simulations performed are described, in section 3 the results of the simulations are shown, and in section 4 a summary and a discussion based on the results are given.

2. Simulations performed

Our simulations are performed with the massively parallel code Nek5000.[11] Nek5000 solves the incompressible Navier–Stokes equations via a Legendre polynomial-based Spectral Element Method (SEM) and is optimized for MPI-based (Message Passing Interface) usage on supercomputers.[12] The present simulations focus on the impulse