Available online at www.sciencedirect.com
ScienceDirect
Procedia IUTAM 14 (2015) 536 – 542
Procedia IUTAM
www.elsevier.com/locate/procedia

IUTAM_ABCM Symposium on Laminar Turbulent Transition

The influence of stratification and slope in mixing layers

Taira, D. E.[a,*], Schettini, E. B. C.[b], Silvestrini, J. H.[c]

[a] *Instituto Federal Sul-rio-grandense (IFSul), Praça vinte de setembro, 455, Pelotas-RS, 96015-360, Brazil*
[b] *Instituto de Pesquisas Hidráulicas, Universidade Federal do Rio Grande do Sul (UFRGS), Av. Bento Gonçalves, 9500, Porto Alegre-RS, 91501-970, Brazil*
[c] *Faculdade de Engenharia, Pontifícia Universidade Católica do Rio Grande do Sul (PUCRS). Av. Ipiranga, 6681, Porto Alegre-RS, 90619-900, Brazil*

Abstract

The purpose of this numerical work is focused on the dynamics of a stably stratified inclined mixing layer. Both effects, stratification and slope, are considered through relevant flow parameters. Chebyshev's approximations and Direct Numerical Simulation (DNS) are used in the context of linear stability analysis for different Richardson numbers and slopes. Two-dimensional temporal and spatial simulations are employed to examine baroclinic layer and the evolution of primary and secondary Kelvin-Helmholtz instabilities. In three-dimensional configuration, only stratification effects are considered. The numerical results show persistence of the translative instability with formation of intense longitudinal vortices highly influenced by the Richardson number.

Selection and peer-review under responsibility of ABCM (Brazilian Society of Mechanical Sciences and Engineering)
Keywords: inclined mixing layer; stable stratification; Direct Numerical Simulation; linear stability analysis.

1. Introduction

Stratified mixing layers develop in the interface of two parallel streams of fluid moving with different velocities and densities. This kind of flows is often found in nature, such as in the atmosphere due to interaction among air currents or in the mixing between fresh and salt water. The buoyancy effect reduces the perturbation growth rate while the slope effect, for instance, due to topographical features, accelerates the developing of instabilities. The competition between both mechanisms results in various types of instabilities depending on mixing layer density difference and inclination. Thus, the transition to turbulence is governed by the competition between inertial and buoyancy forces, which strongly affect the mixing layer longitudinal spreading growth. Previous results of this kind of flows were obtained through laboratory experiments (Browand & Latigo[1] 1979, Thorpe[15] 1987), using linear stability analysis (Hazel[3] 1972, Negretti et al.[10] 2008), or by numerical simulations (Staquet[14] 2001, Smyth[12] 2003, Martinez et al.[9] 2007), among others.

The main objective of this numerical work is to study the stratification and slope influence in stably stratified mixing layers. Direct numerical simulation (DNS) and Chebyshev's approximations are used to quantify the temporal

* Corresponding author. Tel.: +55-53-2123-1020.
E-mail address: davi@pelotas.ifsul.edu.br

2210-9838
Selection and peer-review under responsibility of ABCM (Brazilian Society of Mechanical Sciences and Engineering)
doi:10.1016/j.piutam.2015.03.084

amplification of perturbations of linear problems. For spatially-developing simulations, 2D and 3D configuration domains are considered to follow the spatial evolution of primary and secondary instabilities and three-dimensional vortex structures.

2. Governing equations

The fluid motion governing equations are: continuity, Navier-Stokes in the Boussinesq approximation, and mass transport. In dimensionless, they are stated as,

$$\vec{\nabla} \cdot \vec{u} = 0, \qquad \frac{\partial \vec{u}}{\partial t} + \frac{1}{2}[\vec{\nabla}(\vec{u} \otimes \vec{u}) + (\vec{u} \cdot \vec{\nabla})\vec{u}] = -\vec{\nabla}\Pi + \frac{1}{Re}\vec{\nabla}^2\vec{u} + Ri\,\rho\,\vec{e_\theta}, \qquad \frac{\partial \rho}{\partial t} + (\vec{u} \cdot \vec{\nabla})\,\rho = \frac{1}{RePr}\nabla^2\rho, \tag{1}$$

where $\vec{u} = (u, v, w)$ is the velocity field, $\vec{e_\theta} = (sin\ \theta, -cos\ \theta, 0)$ with the slope given by θ (Fig. 1a), Π is the modified pressure field, and ρ the density. The reference parameters are half velocity difference ($U = (U_1 - U_2)/2$), initial vorticity thickness ($\delta_i = 2U/|\partial\overline{u}/\partial y|_{t=0,y=0}$) and density reference (ρ_0). The Reynolds number, bulk Richardson number and Prandtl number are defined, respectively, by $Re = U\delta_i/\nu$, $Ri = g\Delta\rho\delta_i/(2\rho_0 U^2)$ and $Pr = \nu/\kappa$, where g is the gravitational acceleration, $\Delta\rho$ the density difference, ν the kinematic viscosity and κ the molecular diffusivity.

To perform linear stability analysis, the normal modes method is employed. The non-dimensional governing linear stability equation is given by

$$(\phi_{yy} - \alpha^2\phi) - \frac{u_{yy}}{(u-c)}\phi - Ri\,\frac{\rho_y\ cos\ \theta}{(u-c)^2}\phi - Ri\frac{sin\ \theta}{j\alpha(u-c)^2}\left[\rho_{yy}\phi - \frac{\rho_y u_y}{u-c}\phi + \rho_y\phi_y\right] = 0, \tag{2}$$

where $u(y), \rho(y)$ are the base velocity and density profiles, subscripts y and yy denote differentiation with respect to the vertical direction, respectively, ϕ is the complex disturbance amplitude, $\alpha = \alpha_r$ is the wave-number, $c = \omega/\alpha = c_r + jc_i$ is the complex wave speed, and the amplification rate is defined by $\omega_i = \alpha_r c_i$. Density diffusion and viscous term have been neglected for Eq. (2) development.

3. Initial and boundary conditions

The velocity and density base profiles are given by

$$u(x,y,z,t) = U_C - U\ tanh\left(\frac{2y}{\delta_i}\right), \qquad v(x,y,z,t) = w(x,y,z,t) = 0, \qquad \rho(x,y,z,t) = -\ tanh\left(\frac{2y}{\delta_\rho}\right), \tag{3}$$

where δ_ρ represents the initial density thickness and x, y and z are the streamwise, vertical and spanwise directions, respectively. For temporal simulations, the convection velocity is $U_C = 0$ and initial conditions are $u = u(x, y, t = 0)$, $\rho = \rho(x, y, t = 0)$. Sinusoidal perturbation field (u'_0, v'_0) of maximum amplitude A_f is added to the base velocity profile. Boundary conditions are periodic at $x = 0$ and $x = L_x$, and free-slip at $y = \pm L_y/2$. For spatially-developing mixing layers, the boundary conditions are $u = u(x = 0, y, z, t)$, $\rho = \rho(x = 0, y, z, t)$, $U_1 = 3U$, $U_2 = U$ and $U_C = (U_1 + U_2)/2$ (Fig. 1a). In the inflow boundary condition, the velocity and density profiles (Eq. 3) are used while at the outlet, an outflow boundary condition, $\frac{\partial\varphi}{\partial t} + U_C\frac{\partial\varphi}{\partial x} = 0$ is solved where φ represents u,v or ρ.

4. Numerical methods

The governing equations (Eq. 1) are solved numerically using the `Incompact3d` computational code[5], which is based on compact sixth-order finite difference schemes for spatial differentiation and a second-order Adams-Bashforth scheme for time integration. To treat the incompressibility condition, a fractional step method requires to solve a Poisson equation. This equation is fully solved in spectral space via the use of relevant Fast Fourier Transforms. For three-dimensional simulations, a parallel version of the computational code based on a powerful 2D domain decomposition method is used[6]. The linear stability equation, Eq. (2), is solved via Chebyshev's approximations.