Available online at www.sciencedirect.com

ScienceDirect

Procedia IUTAM 11 (2014) 98 – 109

Procedia IUTAM

www.elsevier.com/locate/procedia

Numerical study of a non-local weakly nonlinear model for a liquid film sheared by a turbulent gas

Te-Sheng Lin[a], Dmitri Tseluiko[a], Serafim Kalliadasis[b]

[a]Department of Mathematical Sciences, Loughborough University, Loughborough, LE11 3TU, UK
[b]Department of Chemical Engineering, Imperial College London, London, SW7 2AZ, UK

Abstract

We investigate a weakly nonlinear equation that arises in the modelling of wave dynamics on a liquid film flowing down an inclined plane when a turbulent gas flows above it. The model is the Kuramoto-Sivashinsky equation with an additional non-local term multiplied by a parameter representing the relative importance of the turbulent gas. The non-local term has a dispersive effect, destabilising effect on long waves and stabilising or destabilising effect on short waves depending on whether the gas flows downwards or upwards. We investigate the influence of this term on the dynamics of the Kuramoto-Sivashinsky equation by extensive numerical experiments. When the gas parameter is sufficiently large, the solution evolves into a row of weakly interacting pulses.

Selection and peer-review under responsibility of scientific committee of Nonlinear Interfacial Wave Phenomena from the Micro- to the Macro-Scale

Keywords:

1. Introduction

Gas-liquid flows occur both in nature and in numerous technological applications such as chemical reactors, cooling systems and evaporators. Here we consider a liquid film that flows down a lower wall of an inclined channel under the action of gravity and with a counter-current turbulent gas flowing above the liquid film. Counter-current gas-liquid flows have been actively studied both experimentally and theoretically starting from the experiments of Semyonov [1], who analysed counter-current flows of water liquid films and air in glass tubes. He found that such flows are characterised by various interesting phenomena of which the most interesting one is the so-called flooding phenomenon: as the gas flow rate is increased, the amplitude of the interfacial waves grows very rapidly before the complete flow reversal.

Other experimental works on counter-current gas-liquid flows include those in Refs. [2,3,4,5,6,7,8,9,10,11,12,13]. As a result of these studies, there have appeared a number of empirical relations that attempt to express the gas velocity at which flooding occurs as a function of physical properties of the gas and the liquid and the geometrical properties of the channel. Theoretical investigations of flooding include works by Shearer and Davidson [14], Guguchkin *et al.* [15], Demekhin [16], Jurman and McCready [17], Peng *et al.* [18]. Trifonov [19,20,21] used an approach in which, under appropriate conditions, the gas problem can be solved independently of the problem for the liquid film following the studies of Miles [22] and Benjamin [23]. Trifonov then analysed the problem for the liquid film using full Navier-Stokes equations. Recently, Tseluiko and Kalliadasis [24] and Vellingiri *et al.* [25] adopted an approach similar to that of Trifonov, but with a more accurate model for the gas phase (which gives significantly better agreement with

Email addresses: t.lin@lboro.ac.uk (Te-Sheng Lin), d.tseluiko@lboro.ac.uk (Dmitri Tseluiko), s.kalliadasis@imperial.ac.uk (Serafim Kalliadasis)

2210-9838
Selection and peer-review under responsibility of scientific committee of Nonlinear Interfacial Wave Phenomena from the Micro- to the Macro-Scale
doi:10.1016/j.piutam.2014.01.052

experimental studies of e.g. Thorsness *et al.* [26] and Zilker *et al.* [27]) and derived a low-dimensional integral-boundary-layer (IBL) model for the liquid film that is more suitable for a systematic investigation of gas-liquid flows than the full Navier-Stokes equations. The IBL model for a free falling film was first introduced by Shkadov [28] and was improved by Ruyer-Quil and Manneville [29,30,31]. Tseluiko and Kalliadasis [24] and Vellingiri *et al.* [25] extended the approach of Ruyer-Quil and Manneville to two-phase gas-liquid flows. These models have been further extended to include additional complexities, e.g. Marangoni effects [32,33,34,35]. The advantage of IBL models is that they capture accurately the instability onset and they correctly describe nonlinear waves sufficiently far away from the critical Reynolds number.

In the present study, we consider a weakly nonlinear model that is valid in a close neighbourhood of the critical Reynolds number and is derived under the assumption that the amplitude of the interfacial waves is small. The model was derived in Ref. [24]. It is the Kuramoto-Sivashinsky (KS) equation with an additional non-local term that represents the effect of the turbulent gas. This non-local term has a dispersive effect, destabilising effect on long waves and stabilising or destabilising effect on short waves depending on whether the gas flows downwards or upwards. It is well known that the dynamics of the KS equation is chaotic in sufficiently large domains, see e.g. Refs. [36,37,38,39]. On the other hand, it has been shown that dispersion in the form of a third-derivative term has a regularising effect on the chaotic dynamics of the KS equation and the solution evolves into arrays of travelling pulses, see e.g. Refs. [40,41,42]. In the more recent studies of Refs. [43,44,45,46] the regularising effect of dispersion was further analysed and a rigorous coherent-structure theory for solitary pulses' interaction was developed; the theory has also been extended to the free falling film problem. In the present study, we analyse how the regularising dispersive effect and destabilising effect of the non-local 'gas term' affect the dynamics of the KS equation.

The paper is organised as follows. In Sec. 2, we present a summary of the derivation of the weakly nonlinear model for the liquid film in the presence of a turbulent gas. In Sec. 3, we present our numerical results. We conclude in Sec. 4.

2. Summary of model derivation

We outline briefly the derivation of a weakly nonlinear model describing wave evolution on a liquid film sheared by a turbulent gas. For more details see Refs. [24,25].

The main idea in the derivation is that under appropriate conditions the gas-liquid interface can be considered as a solid wall for the gas problem. Therefore, for any prescribed interface profile, one can solve the gas problem independent of the liquid flow. Under such an approach, the shear and normal stresses acting by the turbulent gas on the interface can be expressed in terms of the interface profile, and these stresses will enter the stress balance conditions for the liquid problem. Following the well-known long-wave approach, one can derive the evolution equation of the height of the liquid film. A weakly nonlinear expansion of this equation results in a KS equation with additional non-local terms representing the effect of the turbulent gas.

2.1. Gas problem

The gas flow is modelled by the incompressible Renolds-averaged Navier-Stokes equations. We consider the case when the gas flows upwards. The equations are non-dimensionalised using $L_g = \mu_g / \sqrt{\rho_g T_w}$ as the length scale, where ρ_g, μ_g are the density and viscosity of the gas, respectively, and T_w is the magnitude of the shear stress along the wall for the case when the lower wall is flat. The velocity scale is chosen as the so-called friction velocity, $U_g = \sqrt{T_w/\rho_g}$. Besides, T_w is used as the scale for the pressure and the Reynolds stresses. Introducing the stream function Ψ and eliminating the pressure from the governing equations, we obtain

$$\nabla^4\Psi = -\frac{\partial(\Psi, \nabla^2\Psi)}{\partial(x,z)} - \mathscr{R}, \tag{1}$$

where

$$\nabla^2 = \partial_x^2 + \partial_z^2, \quad \nabla^4 = (\nabla^2)^2, \tag{2}$$

and

$$\frac{\partial(f,g)}{\partial(x,z)} = (\partial_x f)(\partial_z g) - (\partial_z f)(\partial_x g), \quad \mathscr{R} = \partial_{xz}\tau_{11} + \partial_z^2\tau_{12} - \partial_x^2\tau_{12} - \partial_{xz}\tau_{22}. \tag{3}$$