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Two-dimensional closed-form model for temperature in living tissues for hyperthermia treatments

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A B S T R A C T

This research article determines an exact analytical expression for 2-D thermal field in single layer living tissues under a therapeutic condition by means of Fourier and non-Fourier heat transfer approaches. An actual spatially dependent initial condition has been adopted to analyze the heat propagation in tissues. The exact analytical determination for this actual initial condition for temperature may be difficult. However, in this study, an approximate analytical method has newly been established for an appropriate initial condition. With this initial expression, an exact temperature distribution for 2-D heat conduction in plane co-ordinates has been investigated for the predefined therapeutic boundary condition to have knowledge for practical aspects of the thermal therapy. Laplace Transform Method (LTM) in conjunction with the Inversion Theorem is used for the analytical solution treatment. We have utilized both Pennes' bioheat equation (PBHE) and thermal wave model of bioheat equation (TWMBHE) for the analysis. The influence of thermo-biological behavior on 2-D heat conduction in tissues has been studied with the variation of several dependable parameters in relation to the Hyperthermia treatment protocol in a moderate temperature range (42–45 °C). The result in the present study has been evidenced for the biological heat transfer for the enforcement of different circumstances and also has been validated with the published value where the maximum temperature deviation of 2.6% has been recorded. We conclude that the temperature curve for TWMBHE model shows a higher waveform nature for low thermal relaxation time and this wavy nature gradually diminishes with an increase in relaxation time. The maximum peak temperature attains 46.3 °C for the relaxation time = 2 s and with the increase in the relaxation time the peak temperature gradually falls. The impact of blood perfusion rate on the relaxation time has also been established in this paper.

1. Introduction

The study of thermal analysis in biological bodies influenced by external heat sources have significantly attracted to the researchers in past two decades due to its importance in clinical science and biomedical engineering specifically thermal therapy (Hyperthermia, Thermal ablation, etc.) treatment of cancer diseases. To evaluate thermal field, most researchers have implemented Pennes' bioheat equation (PBHE) (Pennes, 1948) which is given in Table 1. The extensive utilization of this published model may be due to its simplicity in the solution by several numerical and analytical methods. But as PBHE is based on Fourier's hypothesis which is purely a theoretical prospects (infinite speed of propagation of energy spectrum), exact thermo-biological behavior might not be possible to capture.

In reality, energy propagation takes place in a finite speed as any thermodynamic process possesses steady state condition after a certain period of time lag (relaxation time). This phenomenon is dominant in biological structures as they are highly non-homogeneous and non-uniform. It requires a certain relaxation time to gain sufficient amount of energy for the transmission to a nearest medium. To confine the essence of the finite energy approach, Cattaneo (1958) and Vernotte (1958) concurrently developed C–V model (alternatively single phase lag model) in which τ_q is the thermal relaxation time for heat flux, and based on this fact thermal wave model of bioheat equation (TWMBHE) has been developed (refer Table 1).

Kaminski (1990) suggested theoretical value of relaxation time $\tau_q = 25 - 30s$ for non-homogeneous structures whereas Mitra et al. (1995) observed approximately $16s$ for processed meat by an experimental observation. We have endeavored to glimpse the selected major research outcome in the arena of biological heat transfer influenced by therapeutic treatments. Liu and Xu (1999) estimated exact temperature response in living tissues by employing oscillating heat source with phase shifts. They investigated the influence of thermal contact resistance on the perfusion estimation and it leads to a theoretical

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Nomenclature

A	dimensionless constant introduced in Eq. (8)
B	dimensionless constant, $1 - Q_m^*/\beta^2$
Bi	Biot number, hL/k
c_b	specific heat of blood (J kg^{-1} °C^{-1})
c_p	specific heat of tissue (J kg^{-1} °C^{-1})
F	Fourier number, $\alpha t/L^2$
h	heat transfer coefficient (W-m^{-2} °C^{-1})
i	complex number as referred in Eq. (27)
k	effective thermal conductivity of tissue (W-m^{-1} °C^{-1})
L	length of the tissue in both sides (m)
m, n	non-negative integers in series (0, 1, 2, 3….) used in Eq. (36) and Eq. (37)
N	outward to the normal of the surface (m)
q_m	metabolic heat generation rate (W-m^{-3})
q_s	spatial heat generation rate (W-m^{-3})
Q_m^*	dimensionless metabolic heat generation rate introduced in Eq. (3)
S	surface of a curvature (m^2)
t	time of therapeutic heating (s)
T	local temperature of tissue (°C)
T_b	arterial temperature of tissue (°C)
T_S	initial (steady state) temperature of the tissue (°C)
T_{th}	tissue surface temperature of therapeutic heating (°C)
T_∞	ambient temperature to which living tissue has been exposed (°C)
Ve	Vernotte number, $\sqrt{\alpha\tau_q/L^2}$ defined in Eq. (38)
x	spatial coordinate staring form top of tissue surface (vertical direction) of the tissue depicted in Fig. 1(m)
y	spatial coordinate staring form left of tissue surface (horizontal direction) of the tissue depicted in Fig. 1(m)
X	dimensionless coordinate, x/L
Y	dimensionless coordinate, y/L

Greek letters

α	thermal diffusivity of the tissue, $k/\rho c_p$(m^2s^{-1})
β	dimensionless blood flow parameter, $\sqrt{\omega_b c_b L^2/k}$
ρ	density of the tissue (kg m^{-3})
θ	elevated dimensionless temperature of skin tissue, $(T - T_b)/(T_{th} - T_b)$ see Eq. (10)
θ_C	dimensionless constant temperature mentioned in Eq. (7), $(T_\infty - T_b)/(T_{th} - T_b)$
θ_S	dimensionless steady state temperature, defined in Eq. (3)
τ_q	thermal relaxation time (s)
ω_b	blood perfusion rate (kg m^{-3} s^{-1})
λ , δ	variables used in Eq. (4)
σ	transformed variable declared in Eq. (21)
ϕ , ψ	variables used in Eq. (9)
η, ξ	variables introduced in Eq. (13)
ζ	limiting integral in complex plane expressed in Eq. (27)

foundation of noninvasive modality of blood perfusion. Liu et al. (1999) first developed an analytical solution of TWMBHE by consideration of instantaneous heating at the skin surface and this study revealed the necessity of utilizing the thermal wave behavior of bioheat transfer problems involving high heat flux incident on the skin surface for a short duration. Deng and Liu (2002) produced an outstanding research work by establishing a closed form analytical solution of PBHE by Green functions (GFs) method for different types of heating (constant, step, sinusoidal, point or volumetric) at the skin surface as well as inside the skin tissue. The significance of stochastic heating and metabolic fluctuations to the tissue temperature has also addressed in relation to the thermal therapy, thermal burn injury, and thermal comfort analysis. Moreover, an approach to optimize cancer treatment parameters has been proposed. Shih et al. (2007) examined the thermal response of semi-infinite biological tissues with oscillatory boundary conditions in 1-D form of PBHE and the result has shown the relation between cyclic periods of tissue temperature with imposed sinusoidal heat flux as well as measurement of blood perfusion rate by using phase shift between temperature and heat flux oscillation wave.

Liu (2008) analyzed thermal behavior of living tissue subjected to different heating conditions in 1-D heat transfer by implementing modified discretization scheme in conjunction with the Laplace transform method (LTM) due to difficulties involved in numerical oscillation along with the influence of finite heat propagation and discontinuities involved in time dependent surface heat flux. Xu et al. (2008) presented an elaborated study of non-Fourier thermo-mechanical behavior of skin tissues under various surface heating conditions by the establishment of exact solutions of temperature field, thermal stress and thermal damage field for a single layer skin tissue. Also an important conclusion from this study has been found that the thermal relaxation time for the heat flux τ_q has more dominance in thermal wave propagation in biological systems in comparison to the thermal relaxation time for the temperature gradient τ_T. Tung et al. (2009) proposed modified 1-D 'Hyperbolic heat transfer equation' (HHTE) in skin tissues and outlined the fundamental difference between parabolic and hyperbolic models. He/she had given a specific reason for using HHTE particularly for the applications involved large amount of heat flux at a very short interval.

Table 1
Governing equations of biological heat transfer used for the present analysis.

Model	Postulate
PBHE (Pennes, 1948): $\nabla.(k\nabla T) + \omega_b c_b(T_b - T) + q_m + q_s = \rho c_p \frac{\partial T}{\partial t}$	Fourier hypothesis (Fourier, 1878): $q(\vec{r}, t) = -k\nabla T(\vec{r}, t)$
TWMBHE (Liu et al., 1999): $\nabla.(k\nabla T) + \omega_b c_b(T_b - T) + q_m + q_s + \tau_q\left(-\omega_b c_b\frac{\partial T}{\partial t} + \frac{\partial q_m}{\partial t} + \frac{\partial q_s}{\partial t}\right) = \rho c_p\left(\frac{\partial T}{\partial t} + \tau_q\frac{\partial^2 T}{\partial t^2}\right)$	C-V hypothesis (Cattaneo, 1958, Vernotte, 1958): $q(\vec{r}, t + \tau_q) = -k\nabla T(\vec{r}, t)$

Cotta et al. (2010) introduced a 'generalized integral transform technique' (GITT) for solving 1-D PBHE equation with variable thermo-physical properties of tissue and blood perfusion term in a heterogeneous media as a linear function with the temperature. Ahmadikia et al. (2012) established an analytical solution based on LTM for laser irradiated skin tissue for small and large values of albedo and results have indicated the discrepancy between PBHE and TWMBHE based on the selection of reflection power of laser irradiation and scattering coefficient of tissue. Askarizadeh and Ahmadikia (2014) solved 1-D dual-phase-lag (DPL) model of bioheat transfer in skin tissues with the help of LTM to portray the impact of thermal damage. He concluded that lower heating frequency possesses better penetrative potential by imposing an oscillating heat flux on the skin surface. Kumar et al. (2015) employed a numerical scheme (Finite element wavelet Galerkin method) to study the temperature distribution in living tissues for Hyperthermia treatment. They reported that DPL model proves to be more accurate than other bioheat models in terms of precise prediction of temperature response during Hyperthermia treatments. On the other hand, Kundu (2016) investigated 1-D Fourier and non-Fourier analysis by the 'separation of variables' for different therapeutic surface conditions and an influence of Vernotte number (Ve) and Fourier number (F) on the temperature response has been observed in relation with the