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Three-Dimensional Analysis of Simply Supported Functionally Graded Plate with Arbitrary Distributed Elastic Modulus*

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Abstract: Three-dimensional bending analysis is presented for a simply supported orthotropic functionally graded rectangular plate in this paper. Assuming that material properties have arbitrary variations along the plate-thickness direction, Peano-Baker series solution is obtained for the elastic fields of the functionally graded plate subjected to mechanical loads on its upper and lower surfaces by means of state space method. The correctness of the obtained series solution is validated through numerical examples. The influence of the structural response of the plate is also studied when material properties have different dependence on the thickness-coordinate. The results show that the solution is valid for the material properties of arbitrary dependence on the thickness-coordinate of the plate.

Key words: functionally graded plate; Peano-Baker series; three-dimensional bending analysis; state space method

Introduction

Functionally graded materials (FGMs) are heterogeneous composite materials with gradient compositional variations of the constituents (e.g., metallic and ceramic) from one surface of material to the other which results in continuously varying material properties. This continuously varying composition eliminates interfacial discontinuity, thus, the stress distributions are smooth.

In the past few years, a number of methods have been proposed to analyze the elastic behaviour of FGMs. Praveen and Reddy[1] analyzed the nonlinear static and dynamic response of functionally graded ceramic-metal plates using the first-order deformation theory. Della and Venini[2] developed a hierarchic family of finite elements according to the Reissner-Mindlin theory. Khoma[3] developed a method of constructing a general solution of the equilibrium equations for inhomogeneous transversely isotropic plates with elastic moduli that depend linearly on the transverse coordinate. Sankar[4] obtained an elasticity solution for a simply supported functionally graded plate under cylindrical bending. Zhong and Shang[5] presented a three-dimensional analysis for a rectangular plate made of orthotropic functionally graded piezoelectric material when the plate is simply supported and grounded along its four edges and mechanical and electric properties of the material are assumed to have the same exponent-law dependence on the thickness coordinate. Vel and Batra[6,7] obtained a closed form solutions for three-dimensional deformations of a simply supported functionally graded rectangular plate subjected to mechanical and thermal loads, and for free and forced vibrations of simply functionally graded rectangular plates. Bian et al.[8] developed a plate theory using the concept of shape function of the transverse coordinate to determine the stress distribution in an orthotropic functionally graded plate subjected to cylindrical bending. Martin et al.[9] solved the problem of an

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unbounded, three-dimensional, elastic exponentially graded solid. Pan[10] presented an exact solution for three-dimensional, anisotropic, linearly elastic, and functionally graded rectangular composite laminates under simply supported edge conditions. The solution was expressed in terms of the pseudo-Stroh formalism, and the composite laminates can be made of multilayered FGMs with their properties varying exponentially in the thickness direction. Woo and Meguid[11] developed series solutions for large deflections of functionally graded plate under transverse loading and a temperature field using von Karman theory. Cheng and Batra[12] used an asymptotic expansion method to analyse three-dimensional thermoelastic deformations of functionally graded elliptic plates, rigidly clamped at the edges. Almajid et al.[13] applied a modified classical lamination theory to predict the stress and out-of-displacement of a newly proposed piezoelectric functionally graded bimorph.

In the above studies, most scholars used specific distribution of material properties. However, for arbitrary graded distribution, effective methods are very few. The objective of this work is to present a Peano-Baker series solution of a simply supported functionally graded rectangular plate of arbitrary graded distribution of material properties based on three-dimensional elasticity theory.

1 Formulation of the Problem

Consider an FGM rectangular plate of uniform thickness h, as shown in Fig. 1. Introduce a Cartesian coordinate system $\{x_i\}$ $(i=1,2,3)$ such that the bottom and top surfaces of the undeformed plate lie in the plane $x_3=0$ and $x_3=h$. The functionally graded plate is assumed to have length a and width b in x_1-x_2 plane. In this paper, the Einsteinian summation convention over repeated indices of tensor components is used, with Latin indices ranging from 1 to 3 while Greek indices over 1 and 2.

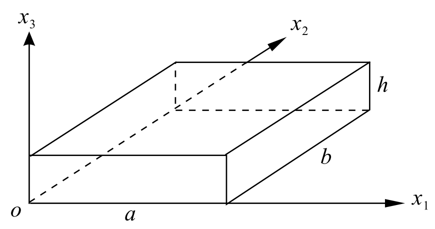

Fig. 1 Sketch of rectangular plate

In the absence of body forces, the field equations of elastic equilibrium is

$$\sigma_{ij,j}=0 \tag{1}$$

where σ_{ij} is the stress tensor, a comma denotes partial differentiation with respect to the coordinate x_i. The strain ε_{ij} is related to the elastic displacements u_i through the following formula:

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{2}$$

The constitutive relationship of FGMs is

$$\sigma_{ij}=c_{ijkl}\varepsilon_{kl} \tag{3}$$

where c_{ijkl} is the elastic stiffness tensor, with the interchanging symmetries $c_{ijkl}=c_{jikl}=c_{ijlk}=c_{klij}$. Unlike in a homogeneous material, c_{ijkl} is now functions of the coordinates x_i ($i=1,2,3$). In most real cases, the material property parameter is varied continuously in one direction, which is assumed to be in x_3 direction for the present analysis.

Next, an orthotropic functionally graded material was considered, for which the nonzero components of the elastic stiffness tensor are c_{1111}, c_{2222}, c_{3333}, c_{1122}, c_{1133}, c_{2233}, c_{2323}, c_{1313}, and c_{1212}. Using state space method, the following relationships in matrix form can be obtained from Eqs. (1)-(3):

$$\partial_3\begin{bmatrix}\boldsymbol{\Pi}\\ \boldsymbol{\Gamma}\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \boldsymbol{A}\\ \boldsymbol{B} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Pi}\\ \boldsymbol{\Gamma}\end{bmatrix} \tag{4}$$

$$\boldsymbol{P}=\boldsymbol{\Phi\Lambda} \tag{5}$$

where $\partial_i=\partial/\partial x_i$, and

$$\boldsymbol{\Pi}=[u_1 \quad u_2 \quad \sigma_{33}]^{\mathrm{T}},\ \boldsymbol{\Gamma}=[\sigma_{13} \quad \sigma_{23} \quad u_3]^{\mathrm{T}} \tag{6}$$

$$\boldsymbol{\Lambda}=[\boldsymbol{\Pi} \quad \boldsymbol{\Gamma}]^{\mathrm{T}},\ \boldsymbol{P}=[\sigma_{11} \quad \sigma_{22} \quad \sigma_{12}]^{\mathrm{T}} \tag{7}$$

The operator matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{\Phi}$ contain the in-plane differential operators ∂_1 and ∂_2, and depend on x_3 only through the material moduli:

$$\boldsymbol{A}=\begin{bmatrix}c_{1313}^{-1} & 0 & -\partial_1\\ 0 & c_{2323}^{-1} & -\partial_2\\ -\partial_1 & -\partial_2 & 0\end{bmatrix} \tag{8}$$

$$\boldsymbol{B}=\begin{bmatrix}-k_1\partial_1^2-c_{1212}\partial_2^2 & k_2\partial_1\partial_2 & -k_3\partial_1\\ k_2\partial_1\partial_2 & -c_{1212}\partial_1^2-k_5\partial_2^2 & -k_4\partial_2\\ -k_3\partial_1 & -k_4\partial_2 & c_{3333}^{-1}\end{bmatrix} \tag{9}$$

$$\boldsymbol{\Phi}=\begin{bmatrix}k_1\partial_1 & k_6\partial_2 & k_3 & 0 & 0 & 0\\ k_6\partial_1 & k_5\partial_2 & k_4 & 0 & 0 & 0\\ c_{1212}\partial_2 & c_{1212}\partial_1 & 0 & 0 & 0 & 0\end{bmatrix} \tag{10}$$

where

$k_1=c_{1111}-c_{1133}^2/c_{3333}$, $k_2=c_{2233}c_{1133}/c_{3333}-c_{1122}-c_{1212}$, $k_3=c_{1133}/c_{3333}$, $k_4=c_{2233}/c_{3333}$, $k_5=c_{2222}-c_{2233}^2/c_{3333}$,