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Steady-State Analysis of Target Tracker with Constant Input/Bias Constraint

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Abstract: Navigation and surveillance applications require tracking constant input/bias targets. When the target's trajectory follows a constant input/bias constraint, model mismatching caused by conventional tracking algorithms can be handled by a delayed update filter (DUF). The statistical convergence and stability properties of the delayed update filter were studied to insure the rationality of its steady-state analysis. A steady-state filter gain was then designed for a constant-gain DUF to reduce the computations without much performance loss. Simulations demonstrate the potential of the constant-gain DUF, and the CGDUF is nearly 60% faster than the DUF without much loss in steady-state tracking accuracy.

Key words: target tracking; Kalman filter; delayed update filter

Introduction

Numerous applications such as environmental monitoring, military surveillance, navigation, and control of moving vehicles require target-tracker algorithms which make use of discrete-time noisy observations to estimate and predict the kinematics of a dynamic target[1]. A significant body of literature exists which addresses the problem of track-while-scan systems. The most ubiquitous recursive estimation technique in target tracking, the discrete-time Kalman filter[2] (KF), models the arrival of an observation as a random process whose parameters are related to the sensor characteristics. However, target tracking with the standard KF can lead to divergence if there is a mismatching of the dynamic model or a lack of input/bias information. Therefore, the variable dimension (VD) filter[3] and multiple model (MM) algorithms[4,5] were developed to handle dynamic model mismatching by modeling changes of the dynamical models. The two-stage filter[6] and the input estimation (IE) method[7] were developed to de-bias the tracking filter through estimates of the system input/bias, and then correct the filter output online. Additionally, a maneuver detector (MD)[7,8] may be needed in some algorithms to judge whether a target is maneuvering and to smoothly adapt the algorithm to a variable dynamic model. Alouani and Blair proposed an effective scheme[9] utilizing a kinematic constraint for tracking a constant speed, maneuvering target, which gives improved performance with little increase in computations. Liu et al.[10] modified these methods for the problem of tracking a target with a constant input/bias constraint, which also aimed to remove the influence caused by the mismatch of common dynamic models.

The target tracker with a constant input/bias constraint was referred to as the delayed update filter (DUF)[10]. As in earlier methods[9,11], the DUF uses a constant input/bias constraint as a pseudo-measurement to update the current states estimated via the KF. Because this constraint can remove the influence caused by the mismatch of common dynamic models, the DUF can mine the steady-state performance when tracking a constant input/bias target while its transient performance is maintained as maneuvers occur. Optimal filtering theory[12] was used to prove

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the asymptotically stable and statistically convergent system. Hence, the rationality of the DUF steady-state analysis can be guaranteed. Numerical solutions of the steady-state filter gains for the DUF are obtained in the second-order model. A constant-gain filter constructed with the steady-state filter gains of the DUF (such as the α-β filter instead of the second-order KF) is developed here as a constant-gain DUF (CGDUF). In practice, target tracking with a CGDUF significantly reduces the computational cost without much performance loss relative to the DUF. Finally, the advantages and effectiveness of the CGDUF are verified in simulations comparing the DUF and CGDUF algorithms with the KF and α-β filter.

1 Problem Formulation

1.1 Delayed update filter

This analysis starts from the discrete Kalman filtering modeling and formulation and then models the constant input/bias assumption as a constraint to the conventional dynamic system. Consider the following discrete-time linear system[13]:

$$\boldsymbol{z}_{k+1} = \boldsymbol{A}\boldsymbol{z}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{C}\boldsymbol{\omega}_k \tag{1}$$

$$\boldsymbol{y}_k = \boldsymbol{D}\boldsymbol{z}_k + \boldsymbol{\upsilon}_k \tag{2}$$

where $\boldsymbol{z}_k \in \mathbf{R}^n$ is the state vector at time k, $\boldsymbol{y}_k \in \mathbf{R}^m$ is the observation vector, $\boldsymbol{u}_k \in \mathbf{R}^l$ is the input/bias vector injected into this dynamic system, $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are matrices that describe the dynamic system patterns, $\boldsymbol{D}$ is the measurement matrix, and $\boldsymbol{\omega}_k \in \mathbf{R}^p$ and $\boldsymbol{\upsilon}_k \in \mathbf{R}^m$ are zero-mean Gaussian random vectors with covariance matrices $\boldsymbol{Q} \geqslant 0$ and $\boldsymbol{R} \geqslant 0$. The input/bias $\boldsymbol{u}_k$ in Eq. (1) is the velocity when $\boldsymbol{z}_k$ contains only the position state of a target but is the acceleration when $\boldsymbol{z}_k$ contains the position and velocity states. For such a target trajectory which follows a constant input/bias constraint, $\boldsymbol{u}_k$ can be assumed to be a constant velocity vector $\boldsymbol{u}_{\mathrm{CV}}$ or a constant acceleration vector $\boldsymbol{u}_{\mathrm{CA}}$ depending on the definition of $\boldsymbol{z}_k$ with the matrices in Eqs. (1) and (2) chosen accordingly.

Since neither $\boldsymbol{u}_{\mathrm{CV}}$ nor $\boldsymbol{u}_{\mathrm{CA}}$ is known by the tracking system but can be estimated, a new state vector is generally obtained by adjoining $\boldsymbol{u}_k$ to $\boldsymbol{z}_k$ as $\boldsymbol{x}_k = \begin{bmatrix} \boldsymbol{z}_k \\ \boldsymbol{u}_k \end{bmatrix}$. Hence, the system model for an augmented KF is often given by

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}\boldsymbol{x}_k + \boldsymbol{G}\boldsymbol{\omega}_k \tag{3}$$

$$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{\upsilon}_k \tag{4}$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are the dynamic matrices, and $\boldsymbol{H}$ is the measurement matrix. Their classical configurations are listed in Table 1[14]. Based on these configurations, either the velocity state in the constant velocity (CV) model or the acceleration state in the constant acceleration (CA) model is intrinsically modeled as a first-order Markov process with additional non-zero variance dynamic noise $\boldsymbol{\omega}_k$. Hence the dynamic noise $\boldsymbol{\omega}_k$ will be iteratively accumulated into state vector. Since neither the velocity state in the CV model nor the acceleration state in the CA model is actually constant, the constant input/bias constraint in the trajectory is always violated. Other tracking algorithms in common use such as the VD and the interactive multiple model (IMM) filter[4] do not rely on this constant input/bias constraint. Therefore, mismatches of dynamic models occur.

Table 1 DUF configurations in one-dimensional coordinates

Configuration	CV model (second-order model)	CA model (third-order model)
States	$\boldsymbol{z}_k = [x_k],\ \boldsymbol{u}_k = [\dot{x}_k],\ \boldsymbol{x}_k = [x_k\ \ \dot{x}_k]^{\mathrm{T}}$	$\boldsymbol{z}_k = [x_k\ \ \dot{x}_k]^{\mathrm{T}},\ \boldsymbol{u}_k = [\ddot{x}_k],\ \boldsymbol{x}_k = [x_k\ \ \dot{x}_k\ \ \ddot{x}_k]^{\mathrm{T}}$
Dynamic matrices	$\boldsymbol{F} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix},\ \boldsymbol{G} = \begin{bmatrix} 0.5T^2 \\ T \end{bmatrix}$	$\boldsymbol{F} = \begin{bmatrix} 1 & T & 0.5T^2 \\ 0 & 1 & T \\ 0 & 1 & 1 \end{bmatrix},\ \boldsymbol{G} = \begin{bmatrix} 0.5T^2 \\ T \\ 1 \end{bmatrix}$
Measurement matrices	$\boldsymbol{H}$=[1 0], $\boldsymbol{H}^{\mathrm{d}}$=[0 1]	$\boldsymbol{H}$=[1 0 0], $\boldsymbol{H}^{\mathrm{d}}$=[0 0 1]
Pseudo-measurement	$y_k^{\mathrm{d}} = \frac{1}{N}\sum_{i=0}^{N-1}\hat{\dot{x}}_{k-i}$	$y_k^{\mathrm{d}} = \frac{1}{N}\sum_{i=0}^{N-1}\hat{\ddot{x}}_{k-i}$
$\boldsymbol{R}_k^{\mathrm{d}}$	$\boldsymbol{R}_k^{\mathrm{d}} = p_{\dot{x}\dot{x},k}$	$\boldsymbol{R}_k^{\mathrm{d}} = p_{\ddot{x}\ddot{x},k}$
Constant DUF gain	$\boldsymbol{K}_\infty = [\alpha\ \ \beta/T]^{\mathrm{T}},\ \boldsymbol{K}_\infty^{\mathrm{d}} = [\alpha^{\mathrm{d}}T\ \ \beta^{\mathrm{d}}]^{\mathrm{T}}$	$\boldsymbol{K}_\infty = [\alpha\ \ \beta/T^2\ \ \gamma/T^2]^{\mathrm{T}},\ \boldsymbol{K}_\infty^{\mathrm{d}} = [\alpha^{\mathrm{d}}T\ \ \beta^{\mathrm{d}}T\ \ \gamma^{\mathrm{d}}]^{\mathrm{T}}$