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Fuzzy Economic Order Quantity Inventory Models Without Backordering*

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Abstract: In economic order quantity models without backordering, both the stock cost of each unit quantity and the order cost of each cycle are characterized as independent fuzzy variables rather than fuzzy numbers as in previous studies. Based on an expected value criterion or a credibility criterion, a fuzzy expected value model and a fuzzy dependent chance programming (DCP) model are constructed. The purpose of the fuzzy expected value model is to find the optimal order quantity such that the fuzzy expected value of the total cost is minimal. The fuzzy DCP model is used to find the optimal order quantity for maximizing the credibility of an event such that the total cost in the planning periods does not exceed a certain budget level. Fuzzy simulations are designed to calculate the expected value of the fuzzy objective function and the credibility of each fuzzy event. A particle swarm optimization (PSO) algorithm based on a fuzzy simulation is designed, by integrating the fuzzy simulation and the PSO algorithm. Finally, a numerical example is given to illustrate the feasibility and validity of the proposed algorithm.

Key words: inventory; fuzzy variable; dependent chance programming; fuzzy simulation; particle swarm optimization

Introduction

Inventory control is an important field in supply chain management. A proper control of inventory can significantly enhance a company's profit. In 1913, the economic order quantity (EOQ) formula was introduced by Harris[1]. Since then, a large number of academic papers have been published describing numerous variations of the basic EOQ model (for a review, see Brahimi et al.[2]). This body of research assumes that the parameters involved in the EOQ model, such as the demand and the purchasing cost, are crisp values or random variables. However, in reality, the demand and the cost of the items often change slightly from one cycle to another. Moreover, it is very hard to estimate the probability distribution of these variables due to a lack of historical data. Instead, the cost parameters are often estimated based on experience and subjective managerial judgment. Thus, the fuzzy set theory, rather than the traditional probability theory, is well suited to the inventory problem.

The fuzzy set theory was first introduced by Zadeh[3], and has now been applied in inventory control systems to model behavior more realistically. In 1981, Sommer[4] used fuzzy dynamic programming to solve a real-world inventory and production scheduling problem, where linguistic statements such as "the stock should be at best zero at the end of the planning horizon" and "diminish production capacity as continuously as possible" were used to describe management's

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fuzzy aspirations for inventory and production capacity reduction in a planned withdrawal from a market. In 1982, Kacprzyk and Staniewski[5] applied the fuzzy set theory to the inventory problem and considered long-term inventory policy-making through fuzzy decision-making models. An algorithm was also presented to find the optimal time-invariant strategy for determining the replenishment to current inventory levels that maximized the membership function for the decision. After that, several scholars developed the EOQ inventory problems in the fuzzy sense. For example, Park[6] examined the economic order quantity model by treating ordering costs and inventory holding costs as trapezoidal fuzzy numbers. The mode and median rules were suggested for transforming the fuzzy cost information into a scalar for input to the EOQ model. Vujošević et al.[7] investigated a fuzzy EOQ model by introducing fuzzy inventory costs and fuzzy order costs. They then obtained the fuzzy total cost and defuzzified the fuzzy total cost with the moments method. Gen et al.[8] expressed their input data as fuzzy numbers, and then the interval mean value concept was introduced to solve the inventory problem. Lee and Yao[9] depicted the order quantity as a triangular fuzzy number, then obtained the fuzzy total cost, and defuzzified it with the centroid method. Yao et al.[10] depicted the order quantity and the total demand as triangular fuzzy numbers, and then solved the problem following the same method as Lee and Yao[9]. Yao and Chiang[11] treated the total demand and the cost of storing one unit per day as triangular fuzzy numbers, then obtained the fuzzy total cost, and defuzzified it with the signed distance method and the centroid method.

In all these EOQ problems, the parameters were assumed to be triangular fuzzy numbers or trapezoidal fuzzy numbers. Therefore, the membership function of the total cost can be calculated easily. However, if the membership function of fuzzy variable is complex, for example, when a trapezoidal fuzzy number and a Gaussian fuzzy number coexist in a model, it is hard to obtain the membership function of the total cost by the methods applied in Refs. [6-11]. Moreover, the moments method, the centroid method, and the signed distance method can only be considered as the ranking function for fuzzy numbers in some special problems. For more general cases, such as where a trapezoidal fuzzy number is divided by a Gaussian fuzzy number, the membership function of the quotient is hard to obtain. Hence, the moments method and centroid method are both difficult to apply. In addition, when the membership function figure is multi-ridged, the α-level cut may include several different intervals, but the signed distance only considers the problem in one interval. Thus, all these methods are unsatisfactory. In this paper, the EOQ problem is investigated by introducing the fuzzy theory to an inventory system. As a general extension of the classical EOQ model, a fuzzy expected value model (EVM) and a fuzzy dependent chance programming (DCP) model are constructed, then an intelligent algorithm is suggested for solving these models.

1 Preliminaries

Let Θ be a nonempty set, $P(\Theta)$ be the power set of Θ, and Pos be a possibility measure. Then the triplet $(\Theta, P(\Theta), \text{Pos})$ is called a possibility space. Let A be a set in $P(\Theta)$. The necessity measure of A can then be represented by

$$\text{Nec}\{A\} = 1 - \text{Pos}\{A^c\} \tag{1}$$

where A^c is the complement of A.

Definition 1[12] Let $(\Theta, P(\Theta), \text{Pos})$ be a possibility space, and A a set in $P(\Theta)$. Then the credibility measure of A is defined by

$$\text{Cr}\{A\} = \frac{1}{2}(\text{Pos}\{A\} + \text{Nec}\{A\}) \tag{2}$$

Definition 2 A fuzzy variable ξ is defined as a mapping from a possibility space $(\Theta, P(\Theta), \text{Pos})$ to the set of real numbers, and its membership function is defined by

$$\mu_\xi(x) = \text{Pos}\{\theta \in \Theta \mid \xi(\theta) = x\}, \quad x \in \mathbf{R} \tag{3}$$

Proposition 1[13] Suppose that $(\Theta_i, P(\Theta_i), \text{Pos}_i)$, $i = 1, 2, ..., n$, are possibility spaces. Let $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n = \prod_{i=1}^{n} \Theta_i$, and

$$\text{Pos}\{A\} = \sup_{(\theta_1, \theta_2, ..., \theta_n) \in A} \text{Pos}_1\{\theta_1\} \wedge \text{Pos}_2\{\theta_2\} \wedge \cdots \wedge \text{Pos}_n\{\theta_n\}$$

for each $A \in P(\Theta)$. Then the set function Pos is a possibility measure on $P(\Theta)$, and $(\Theta, P(\Theta), \text{Pos})$ is a possibility space (called the product possibility space of $(\Theta_i, P(\Theta_i), \text{Pos}_i)$, $i = 1, 2, ..., n$).

Definition 3[13] The fuzzy variables $\xi_1, \xi_2, ..., \xi_n$ are said to be independent if and only if

$$\text{Pos}\{\xi_i \in B_i, i = 1, 2, ..., n\} = \min_{1 \leqslant i \leqslant n} \text{Pos}\{\xi_i \in B_i\} \tag{4}$$

for any sets $B_1, B_2, ..., B_n$ of $\mathbf{R}$.