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FM-BEM Evaluation for Effective Elastic Moduli of Microcracked Solids*

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Abstract: A fast multipole boundary element method (FM-BEM) was applied for the analysis of microcracked solids. Both the computational complexity and memory requirement are reduced to $O(N)$, where N is the number of degrees of freedom. The effective elastic moduli of a 2-D solid containing thousands of randomly distributed microcracks were evaluated using the FM-BEM. The results prove that both the differential method and the method proposed by Feng and Yu provide satisfactory estimates to such problems. The effect of a non-uniform distribution of microcracks has been studied using a novel model. The numerical results show that the non-uniform distribution induces a small increase in the global stiffness.

Key words: fast multipole; boundary element; microcrack; effective modulus

Introduction

Generally speaking, brittle or quasi-brittle materials contain large numbers of pre-existing microcracks. Various aspects of the deformation and failure behavior of such materials are associated with the distribution and interaction of these microcracks. Because of the requirement for boundary-only discretization and due to its semi-analytical nature, the boundary element method (BEM) is recognized as a powerful tool for fracture mechanics analysis[1]. However, the dense and asymmetric coefficient matrix makes the conventional BEM inefficient for large-scale problems.

To improve run-time speed and memory storage efficiency, the fast multipole method (FMM)[2] has been applied to the BEM. The fast multipole BEM (FM-BEM) uses the same discretization as the conventional BEM, but uses a quad-tree (for 2-D cases) or an octal-tree (for 3-D cases) for computation and storage. The matrix-vector product is obtained by recursive operations on the tree structure without explicitly forming the coefficient matrix. In recent years, FM-BEM algorithms and their applications have been investigated by many researchers[3-6]. Among these, several schemes have been successfully applied to large-scale fracture analysis[7-12].

In the present paper, an FM-BEM based on dual boundary integral equations (DBIE)[13] is applied for the simulation of a 2-D microcracked solid. By combining multipole expansions with local expansions, both the computational complexity and memory requirement are reduced to $O(N)$, where N is the number of degrees of freedom (DOFs). The effective elastic moduli of 2-D solids containing thousands of randomly distributed microcracks are evaluated using the model, and the numerical results are compared with the corresponding solutions from various micromechanical models. In addition, a novel model is presented for the simulation of a non-uniform distribution of microcracks. The effect of a non-uniform distribution on the effective elastic moduli is evaluated using the FM-BEM scheme.

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1 Formulation and Implementation of the FM-BEM

1.1 DBIE formulation for crack analysis

Figure 1 shows a crack Γ_c including two coincide surfaces, Γ_c^- and Γ_c^+, in a 2-D elastic solid Ω surrounded by an external boundary Γ_o. In the absence of body forces, the following displacement integral equation can be obtained.

$$c_{\alpha\beta}(\boldsymbol{x})u_\beta(\boldsymbol{x}) = \int_{\Gamma_o \cup \Gamma_c^+ \cup \Gamma_c^-} \left(U_{\alpha\beta}(\boldsymbol{x},\boldsymbol{y})t_\beta(\boldsymbol{y}) - T_{\alpha\beta}(\boldsymbol{x},\boldsymbol{y})u_\beta(\boldsymbol{y})\right)\mathrm{d}\Gamma(\boldsymbol{y}) \tag{1}$$

where $u_\beta(\boldsymbol{y})$ and $t_\beta(\boldsymbol{y})$ are the displacement and traction components at field point $\boldsymbol{y}$, $c_{\alpha\beta}(\boldsymbol{x})$ is a free term depending on the shape of the boundary, and $U_{\alpha\beta}(\boldsymbol{x},\boldsymbol{y})$ and $T_{\alpha\beta}(\boldsymbol{x},\boldsymbol{y})$ represent the Kelvin displacement and traction fundamental solutions.

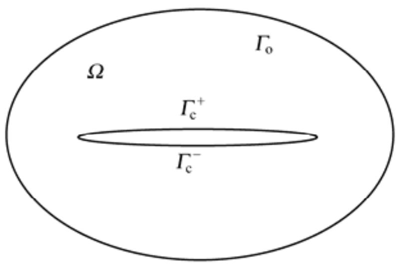

Fig. 1 A 2-D elastic solid with a single crack

Assuming continuity of both strains and tractions at $\boldsymbol{x}$ on a smooth boundary, the following traction boundary integral equation can be obtained.

$$\frac{1}{2}t_\beta(\boldsymbol{x}) = n_\alpha(\boldsymbol{x}) \bullet \int_{\Gamma_o \cup \Gamma_c^+ \cup \Gamma_c^-} \left(D_{\gamma\alpha\beta}(\boldsymbol{x},\boldsymbol{y})t_\gamma(\boldsymbol{y}) - S_{\gamma\alpha\beta}(\boldsymbol{x},\boldsymbol{y})u_\gamma(\boldsymbol{y})\right)\mathrm{d}\Gamma(\boldsymbol{y}) \tag{2}$$

where $n_\alpha(\boldsymbol{x})$ denotes the component of the outward unit normal at $\boldsymbol{x}$. The detailed expressions for the kernel functions $D_{\gamma\alpha\beta}(\boldsymbol{x},\boldsymbol{y})$ and $S_{\gamma\alpha\beta}(\boldsymbol{x},\boldsymbol{y})$ can be found in Ref. [13].

Equations (1) and (2) form the DBIE formulation for crack analysis. The displacement equation (1) is applied for collocation on the external bosundary and on one crack surface; the traction equation (2) is used for collocation on the other crack surface.

1.2 Outline of the FM-BEM

As shown in Fig. 2, the FMM algorithm comprises the following four key operations[2,4].

(1) Multipole expansion. In Fig. 2, a square cell A, whose center is $\boldsymbol{y}_0$, contains a boundary segment Γ_A. Using the multipole expansion, the contribution of the boundary integral on Γ_A with respect to the source point $\boldsymbol{x}$ is concentrated at $\boldsymbol{y}_0$.

(2) Multipole to multipole translation (M2M). Using this operation the contribution of $\boldsymbol{y}_0$ is shifted to point $\boldsymbol{y}_1$, which is the center of A's parent super cell B.

(3) Multipole to local translation (M2L). Using this operation the contribution of $\boldsymbol{y}_1$ is shifted to $\boldsymbol{x}_0$, which is the center of cell C containing $\boldsymbol{x}$.

(4) Local to local translation (L2L). Using this operation the contribution of $\boldsymbol{x}_0$ is shifted to $\boldsymbol{x}_1$, which is the center of child cell D containing $\boldsymbol{x}$.

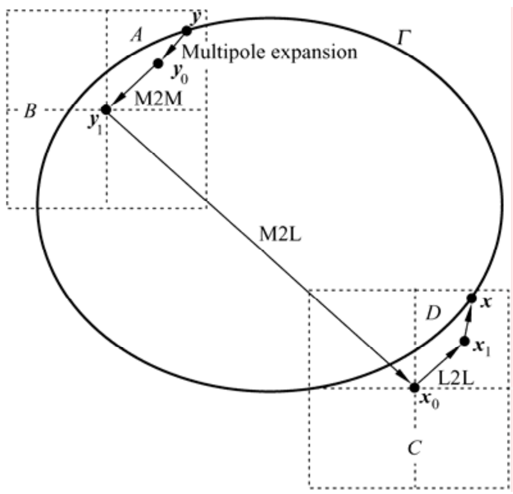

Fig. 2 Key operations of FMM

A detailed description of the above four operations can be found in Refs. [11,12]. In the implementation of the FM-BEM, the boundaries and microcracks are first discretized into elements in the same way as in the conventional BEM. Next, an adaptive tree data structure is constructed for storage and computation. The root of the tree, which is at level 0, is a square cell containing all the boundaries and microcracks of the model. The root is divided into four child cells at level 1. Each child cell is divided in the same way until the number of elements in it is less than a predefined number. Finally, an iterative process is executed to solve the equation system derived from DBIE. In each