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Three-Dimensional Stress and Stress Intensity for Tensioned Flat Plates with Edge Cracks*

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Abstract: The stress in the thickness direction is an important factor influencing the fracture behavior of structural members. A stress σ_y tensioned flat plate with edge cracks is widely used as an analysis model. The stresses σ_x and σ_y for the plate model can be acquired from Neuber's solution. However, the solution is applicable only for a perfect plane stress or plane strain state. As a consequence of the thickness of the plate a three-dimensional (3-D) stress state will arise near the crack tip, resulting in a variation of the distribution of σ_x and σ_y stresses. A full analysis for the 3-D stress fields for a tensioned flat plate with edge cracks has been therefore carried out. The results show that the 3-D stress field near the crack tip is mainly determined by two factors: the thickness of the plate and the curvature radius at the crack tip. A further analysis has been carried out for the stress intensity near the crack tip. In this paper we give some equations matching to the 3-D stress and stress intensity, which describe precisely the stress state near the crack tip, and which can be applied effectively in engineering analysis.

Key words: three-dimensional stress; stress in the thickness direction; stress intensity; crack tip

Introduction

There is a strong relationship between the fracture behavior of a steel structure and the stress field near the crack tip. A very high stress concentration[1-3] occurs in the area near any cracks or holes and the small area near a crack tip is often in a three-dimensional tensile stress state[4]. The tensile stress σ_z in the thickness direction will restrain the plastic flow in the plane at the crack tip and thereby making brittle fracture of large scale members easier to occur[5]. At the same time, the yield strength perpendicular to the crack at the crack tip will increase due to the existence of σ_x and σ_z, namely, the stress intensity. The extra stress intensity, at the crack tip will increase ductile-brittle transition temperature, making fracture more likely[6].

Research about the three-dimensional stress near crack tips has been carried out using many methods. Due to the complexity and difficulty of this problem, the finite element method is an effective technique that has been used widely[7]. Finite deformation photoelasticity has also been applied to the study[8] although more and more are known about the stress state at a crack tip, most results are qualitative, and there are no practical quantitative rules for predicting the three-dimensional stress and stress intensity that could be applied in engineering design. Based on existing achievements, in this paper we report an

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analysis of the three-dimensional stress at a crack tip and of the stress intensity for tensioned plates with edge cracks.

1 Neuber's Solution

1.1 Basic model and solution

The stress and strain of real structures and the cracks in real structures are complicated, so many simple models have been adopted to study the influence of cracks. Much research has shown that the influence of the crack is restricted to the small area near the crack, and that the stress and strain states near the crack are the same in most aspects. The results obtained from a simple model can therefore be applied widely to various structures. A tensioned plate with edge cracks (Fig. 1) is a common model, for which Neuber has given the theoretical solution.

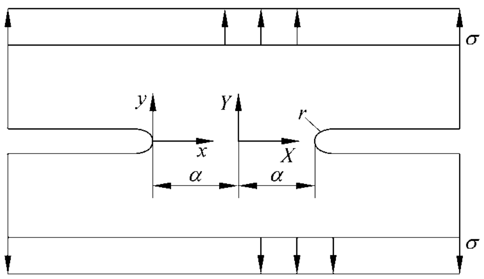

Fig. 1 Tensioned plate with edge cracks

In Cartesian coordinates *X*-*Y* with the origin being the center of the plate, Neuber's solution is

$$\sigma_x = K\sigma_{\mathrm{n}}(w^2-1)/(2w^3) \quad (1)$$

$$\sigma_y = \sigma_1 = K\sigma_{\mathrm{n}}(w^2+1)/(2w^3) \quad (2)$$

where σ_{n} is the nominal stress, $w = \cos V / \cos V_0$. Here, $\mathrm{tg}V_0 = \sqrt{\alpha / r}$, $\sin V = X \sin V_0 / \alpha$. The value of α is half of the net width of the plate in the crack section, and r is the radius of the curvature of the crack tip. K is the stress concentration factor.

1.2 Further research

To simplify Neuber's solution, the origin of the coordinate can be moved to the crack tip, so the new coordinate system transfers from *X*-*Y* to *x*-*y*. The crack studied is very sharp, and its curvature radius r is far smaller than α, i.e., $r \ll \alpha$. For analysis of the fracture of members, only the small area near the crack tip is closely related, so $x \ll \alpha$. Based on these two conditions, Neuber's solution can be simplified as[4]

$$\sigma_x = K\sigma_{\mathrm{n}} j / (1+2j)^{3/2} \quad (3)$$

$$\sigma_y = K\sigma_{\mathrm{n}}(1+j)/(1+2j)^{3/2} \quad (4)$$

where $j = x/r$. Compared with the initial solution, the simplified solution is more transparent. In the area near the crack tip, the simplified solution has a fine precision. A comparison shows that the difference between the initial solution and the simplified solution is only 1.2% in the range of 0-10j from the crack tip.

The stress σ_z in the thickness direction can be expressed as a function of σ_x and σ_y.

$$\sigma_z = m(\sigma_x + \sigma_y) \quad (5)$$

where m is the plane strain factor, which takes a value between 0 and μ (μ is Poisson ratio of the material, the value for structural steels is 0.3). For a plane stress condition, the factor m=0, whereas $m=\mu$ for a plane strain condition. Generally, the limit of σ_z in the thickness direction is

$$0 \leqslant \sigma_z \leqslant \mu(\sigma_x + \sigma_y) \quad (6)$$

The distribution of σ_z decreases away from the crack tip. When the plate is relatively thin, the value of σ_z is small and the stress field near the crack tip is close to the plane stress state. With increasing plate thickness, the stress field near the crack tip gradually shifts into a plane strain condition.

2 Three-Dimensional Stress near the Crack Tip

Many factors are related to the value and distribution of σ_z near the crack tip. A merely qualitative analysis of these relations is not useful for structural design. The objective of the further research on the three-dimensional stress near the crack tip is therefore to acquire quantitative rules and relations between the stress σ_z and these related factors.

2.1 Finite element model (FEM)

Finite element calculations have been carried out using the ANSYS software. The material is set as a linear elastic material with Young modulus E=2.06×10^5 N/mm^2 and Poisson ratio μ=0.3. The model is constructed using solid elements allowing large displacements. The type of solid element used