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Transient Flow in Rapidly Filling Air-Entrapped Pipelines with Moving Boundaries*

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Abstract: A mathematical model is presented for transient flow in a rapidly filling pipeline with an entrapped air pocket. The influence of transient shear stress between the pipe wall and the flowing fluid is taken into account. A coordinate transformation technique is employed to generate adaptive moving meshes for the multiphase flow system as images of the time-independent computational meshes in auxiliary domains. The method of characteristics is used to reduce the coupled nonlinear hyperbolic partial differential equations governing the motion of the filling fluid, entrapped air, and blocking fluid to ordinary differential equations. Numerical solution of resulting equations shows that the transient shear stresses have only a small damping effect on the pressure fluctuations. The peak pressure in the entrapped air pocket decreases significantly with increasing initial entrapped air volume, but decreases slightly with increasing initial entrapped air pressure.

Key words: moving meshes; transient flow in pipelines; entrapped air; transient shear stress; method of characteristics

Introduction

Transient fluid flow in rapidly filling undulating pipelines with entrapped air is of practical interest in many areas, including water supply systems, sewer systems, and oil pipelines, and has received an increasing amount of attention recently[1-3]. A brief review of pipe transient flow with entrapped air is presented by Izquierdo et al.[4]

A theoretical analysis was conducted of unsteady motion of a lengthening water column filling an empty pipeline with an undulating elevation profile[5]. The solution revealed that the early phase of the filling process is characterized by very rapid acceleration followed by less rapid deceleration of the water column. This analysis was later extended to multiphase flow by incorporating the influence of entrapped air between two blocking water columns into the pipe filling process[4,6]. The motion of the water column coupled with the uniform gas compression was analyzed. The effect of entrapped air in a rapidly filling horizontal pipe sealed or outfitted with an orifice at one end has been investigated experimentally and theoretically[2,3]. The cushioning effects of the air pockets prevent the water column from generating high water hammer pressures. All of these investigations are confined to studying the flow behavior in filling pipelines with entrapped air based on the assumption of elastic system behaving as a rigid one with negligible convective acceleration,

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termed the rigid model. This assumption which reduces the nonlinear, first order hyperbolic partial differential equations (PDEs) to ordinary differential equations (ODEs) is reasonable only when the hydraulic condition of the system is slightly unsteady and not greatly influenced by any significant inertia[7].

In addition to this simplification, the friction between the pipe wall and the fluid is accounted for the assumption that the wall shear stresses have a quasi-steady behavior. The influence of transient shear stress on the pipe filling process in the presence of entrapped air has remained up to now unexplored, even though in many applications, fast pipe transient flow occurs after air has been admitted into the pipe in order to avoid cavitations and/or air pockets developing at high points in an intermittent pipe flow system.

Numerical solutions of the transient flow in pipelines with entrapped air are complicated by the fact that the meshes for the liquid flow coupled with isentropic gas compression in a time varying computational domain must be modified during the computation in order to accommodate the changes in the geometry. Also, the pressure wave exhibits sharp front structures generated from the rapid valve openings. Many efforts have been devoted to simulations of transient flow problems with sharp front structures using a moving mesh approach based on the finite element method (FEM)[7,8], finite volume method (FVM)[9], method of lines (MOL)[10], and method of characteristics (MOC)[11]. For transient flows generated by rapid valve openings, MOC is more feasible to solve the nonlinear hyperbolic PDEs[10, 12]; hence it is employed in this paper.

Our aim is to develop a generalized elastic model which includes the effects of transient shear stresses and isentropic air compression on the hydraulic transients to predict potential risks with air-entrapped-pipelines. Time varying liquid domain coupled with isentropic gas compression accounts for fluid-gas interface movement and tracks pressure wave propagation in both the gas and liquid regions. As the presence of two or more blocking fluid columns does not influence the maximum pressure of the first air pocket[4], we consider a simple case with just one entrapped air pocket. In this paper, we restrict ourselves to the analysis of the transient flow problem in a one-dimensional coordinate system. Three coordinates are introduced to transform the momentum and continuity equations governing the motion of fluid and the nonuniform compression of gas in time varying physical domains into equations in time independent computational domains. By employing MOC for the resulting equations, a general numerical formulation has been established which is capable of analyzing the hydrodynamic problem of adjoining liquid and air regions with time varying domains, and hence exploring the effect of entrapped air on the transient flow behavior. The influence of time-dependent shear stress on the transient flow in air entrapped-pipelines is also investigated.

1 Formulation of the Problem

Consider a filling process in an undulating pipeline with an entrapped air pocket, as shown in Fig. 1. A discharge valve positioned upstream is opened and the resultant flow is induced by the negative pressure gradients, followed by decompressing the entrapped air and consequently driving blocking fluid to flow downstream; this filling process is formulated one-dimensionally.

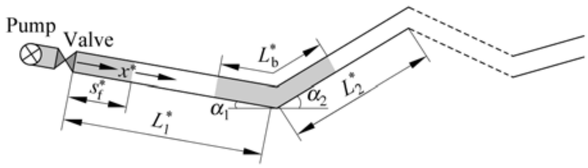

Fig. 1 Schematic of the system under investigation

Fluid motion is described by the velocity $V_k^*(x^*,t^*)$ and pressure $p_k^*(x^*,t^*)$ in which subscript $k=\mathrm{f},\mathrm{b}$ represents the filling and blocking liquids. The expansion and compression of compressible gas is described by the velocity $V_\mathrm{g}^*(x^*,t^*)$ and wave speed $a_\mathrm{g}^*(x^*,t^*)$ (or pressure $p_\mathrm{g}^*(x^*,t^*)$). x^* denotes the coordinate along with the pipeline and t^* denotes time; variables with and without asterisks represent dimensional and dimensionless quantities. The origin is coincident with that of the valve, while the positions of filling fluid-entrapped air and blocking fluid-entrapped air interfaces are denoted by s_f^* and s_b^*. It is assumed that the pipe remains full during transient flow so that there is a liquid column with a well-defined front.

1.1 Fluid domain

Fluid motion is governed by the equation of continuity