Forest Ecology and Management 289 (2013) 404–408

Contents lists available at SciVerse ScienceDirect

Forest Ecology and Management

journal homepage: www.elsevier.com/locate/foreco

Comparison of maximum size–density relationships based on alternate stand attributes for predicting tree numbers and stand growth

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ARTICLE INFO

Article history:
Received 9 July 2012
Received in revised form 19 October 2012
Accepted 22 October 2012
Available online 28 November 2012

Keywords:
Stand density
Self-thinning
Mortality
Growth and yield
Loblolly pine
Pinus taeda

ABSTRACT

Quantification of site occupancy or stand density is essential for modeling forest stand mortality, growth and yield. A variety of measures of stand density have been proposed. Diverse tree and stand attributes have been employed, but direct comparisons of the effectiveness of various measures for estimating number of trees for fully stocked stands and for predicting growth for stands at different stages of development are not possible due to the varying forms of the measures.

Maximum size–density relationships are a widely-used type of stand density measure. Reineke's stand density index (SDI), which is based on number of stems per unit area and quadratic mean stem diameter, is the most commonly-applied stand density measure of this type. In the study reported here, stand density indices were developed by employing the structure of Reineke's index but using the stand attributes of mean stem volume and mean stand height, as well as mean stem diameter. When estimating number of trees and periodic volume growth per unit area using data from sample plots in planted stands of loblolly pine, the SDI based on mean diameter performed best with mean stem volume being second best and mean stand height third.

1. Introduction

Simple and effective indices of competition in forest stands are essential for modeling mortality, growth and yield. The simplest measure of density is number per unit area, but to be useful in quantifying crowding or competition for space, tree size is an essential component of stand density measures. Number of trees is necessary but not sufficient to adequately describe stand density. As individual trees grow in size their demands on site resources and growing space increase. When resources are no longer adequate to support additional growth of all the trees present, self-thinning will be initiated and the number of trees per unit area will decrease. Several indices have been developed to study the influence of density on self-thinning; the indices combine an expression of the size of the average tree (diameter, height, or volume) with the number of trees per unit area. The best known and most commonly employed of these self-thinning or maximum size–density relationships are those of Reineke (1933), Yoda et al. (1963), and Hart (1926). Reineke's stand density index is based on the relationship between numbers of trees per unit area and the quadratic mean diameter of stands, whereas Yoda et al. related mean plant volume (or biomass) to numbers per unit area. Hart proposed a measure based on the average distance between trees and the average height of the dominant canopy. These and other stand density measures are discussed in detail in Burkhart and Tomé (2012, Chapter 8).

The self-thinning rules of Reineke and Yoda et al. assume a common slope between the logarithm of size and the logarithm of density for a wide variety of species in fully-stocked stands. Subsequent investigations, however, have shown that the slope of the self-thinning line varies by species (Pretzsch and Biber, 2005), site index (Bi, 2001), and management inputs, such as initial planting density (VanderSchaaf and Burkhart, 2012). For loblolly pine, the species reported on here, the allometric constant for Reineke's rule has often been taken to be −1.605 as originally reported, but Cao et al. (2000) cited published values ranging from −1.505 to −1.707. Estimates of the slope of the log–log relationship depend on the sample data included and the statistical model used in estimation. Using data from a loblolly pine spacing trial and applying three statistical techniques to estimate the slope of Reineke's maximum size–density relationship for the full data set and for subsets of the data, VanderSchaaf and Burkhart (2007) obtained estimates ranging from −1.0330 to −2.1240; they recommended that an exponent of −1.6855 be used.

Most studies of self-thinning of even-aged, monospecific stands of plants have been based on empirical observations. Various theoretical models have been proposed to explain self-thinning mechanisms, but, after reviewing assumptions of the various models, Reynolds and Ford (2005) opined that the models are inadequate

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http://dx.doi.org/10.1016/j.foreco.2012.10.041

to allow consensus on the processes driving variation in density-dependent mortality and self-thinning.

Although we lack a good understanding of the processes that drive competition-induced mortality, the relationships of Reineke, Yoda et al. and Hart have found application in forest management decisions involving control of density of growth stock. The rules of Reineke and Yoda et al. have been widely applied in the development of density management diagrams to provide guidance for controlling stand density to meet specified management objectives (examples of density management diagrams include Dean and Jokela (1992), Williams (1994), Jack and Long (1996), Sharma and Zhang (2007), and VanderSchaaf and Burkhart (2012)). The height-based relationship of Hart has been applied to develop thinning schedules (Wilson, 1979).

Growth models for even-aged forest stands typically incorporate species, age, site quality (generally based on the site index concept), a measure of stand density, and management treatments. For purposes of prediction, the stand density expression included in growth predictions should result in strong multiple correlation with growth in the presence of stand age and site index.

Maximum size–density or self-thinning relationships based on tree stem diameter, tree volume, and tree height have been promulgated. However, formulations of the various measures are somewhat different. Reineke's index relates number of trees to mean diameter, Yoda's rule is based on mean tree volume versus number of trees, whereas the Hart index is a ratio of mean distance between trees to stand height. In a series of papers Zeide (1985, 1987, 1991, 1995, 2002, 2005, 2010) analyzed self-thinning and stand density using reasoning and empirical evidence and concluded that measures based on diameter are to be preferred. A brief summary of the arguments and conclusions presented by Zeide follows.

Tree stem volume is a principal variable for many forestry purposes, but it is not the best representation of crowding (Zeide, 2010). Crowding depends on the space trees occupy, which is closely related to crown size, not stem size. The size of crowns increases with stem diameter, but, given trees of the same diameter, decreases with tree height within a stand. As elaborated by Zeide (2002):

> "This means that taller trees have smaller crowns than shorter trees with the same diameter. On the other hand, stem volume increases with both stem diameter and height. This reasoning explains why stem diameter ... has a closer relationship with crown size than stem mass or volume."

Zeide (2010) noted that in dense stands with complete crown closure, the number of trees is inversely related to the square of average crown diameter. Therefore, he argued, the variable most closely related to crown diameter will be the best predictor of self-thinning. Average height is not as highly correlated with crown width as average diameter, and stem volume, which involves both diameter and height, is intermediate in correlation. Based on reasoning advanced by Zeide and on empirical relationships between crown width and tree variables, one would expect stand density measures using stem diameter to be "best" followed by those employing stem volume and measures involving height.

The purpose of this investigation was to evaluate the explanatory value of stem diameter, height, and volume for estimating tree numbers in self-thinning stands and for predicting growth in stands of varying ages, site indices, and densities. A common model framework was employed for the three size measures when fitted with data from a set of permanent plots established in loblolly pine plantations across the southeastern United States.

2. Data

The data analyzed in this study came from unthinned plots located in 186 loblolly pine (*Pinus taeda* L.) plantations throughout the Piedmont and Coastal Plain physiographic regions in the southeastern United States. At each plantation included in this thinning study, an unthinned control plot, typically 0.04 ha in size, was established, and all trees located within the plot were measured for dbh and height at the time of plot establishment and at 3-year intervals thereafter. Table 1 contains a summary of plot statistics at the time of initial measurement. Burkhart et al. (1985) provide a full description of the study installation.

Table 1
Summary statistics for the 186 control (unthinned) sample plots at time of study establishment in loblolly pine plantations.

Variable	Minimum	Mean	Maximum
Age (years from planting)	8	15.2	25
Number of planted loblolly surviving (trees ha^{-1})	679	1378	2346
Arithmetic mean dbh of planted loblolly (cm)	6.9	14.5	23.9
Loblolly basal area (m^2 ha^{-1})	5.3	24.1	53.0
Site index (m, base age 25)	13.6	20.0	26.7

3. Analyses and results

3.1. Number of trees

Reineke (1933) noted that in fully stocked even-aged stands the limiting relationship between the number of trees per unit area (N) and the quadratic mean dbh ($\bar{d}_q$) is linear on the log–log scale. For any given $\bar{d}_q$ there is a limit to the number of trees per unit area that can be carried. Further, Reineke found that for a variety of species the intercept varied but the slope of the limiting line was approximately -1.6 on the log–log scale, that is[1]:

$$\log N = -1.6 \log \bar{d}_q + k_1 \quad (1)$$

where k_1 is a constant varying by species and log indicates logarithm.

The so-called 3/2 rule of self-thinning, like Reineke's stand-density index, is based on the concept of a maximum mean size–density relationship. By convention, however, for the 3/2 rule of self-thinning the logarithm of mean tree volume or weight is plotted against the logarithm of the number of trees per unit area. For pure, even-aged stands that are sufficiently crowded such that competition-induced mortality ("self-thinning") is occurring the slope of the line of logarithm of mean volume (or weight) versus logarithm of trees per unit area has been found to be approximately $-3/2$, but the intercept varies by species. That is,

$$\log \bar{v} = -3/2 \log N + k_2 \quad (2)$$

where $\bar{v}$ is the mean tree volume, and k_2 is a constant varying with species.

The relationship between number of trees per unit area and average stem volume could, of course, be expressed with log N on the y-axis and log $\bar{v}$ on the x-axis.

Relative spacing (RS), a commonly-used maximum-size density relationship based on height, is defined as the average distance between trees divided by the average height of the dominant canopy. Assuming square spacing, the average distance between trees can be computed as the square root of the number of m^2 per ha (10,000) divided by the number of trees per ha. The average

[1] In his original paper Reineke (1933) used a value of -1.605 for the common slope; in this discussion the slope value was rounded to -1.6.