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Connection between elastic and electrical properties of cortical bone

X. Gao [a], I. Sevostianov [b,*]

[a] Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, Loughborough, UK
[b] Department of Mechanical and Aerospace Engineering, New Mexico State University, Las Cruces, USA

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ABSTRACT

The paper focuses on the connection between elastic and electrical properties of cortical bone. Both these properties are governed by microstructure that consists of several pore systems filled with mechanically soft and electrically conductive tissue. Microstructural changes induced by aging, various diseases, microgravity conditions etc. lead to variation in both properties. The paper address the problem of evaluation of the changes in mechanical performance (decrease in Young's moduli) via monitoring electrical conductivity. The theoretical results are verified experimentally.

1. Introduction

We establish cross-property connections between mechanical and electrical properties of cortical bone. Such connections relate changes in different physical properties caused by the presence and/or development of certain microstructure. To the best of our knowledge, existence of the cross-property connections has been first recognized by Bristow (1960) who considered metals containing multiple randomly oriented microcracks. The existence of *explicit quantitative cross-property connections* between two physical properties depends on the possibility to express them in terms of the same or sufficiently similar microstructural parameter (Kachanov and Sevostianov, 2005). Critical review of the existing results on cross-property connections with detailed analysis of various applications is given by Sevostianov and Kachanov (2009).

In the context of properties of bone, the observations of cross-property connections are mostly of *qualitative* nature (see, for example, Sierpowska et al., 2006 and Sevostianov, 2014). The main challenge in developing the *quantitative* cross-property connections for cortical bone is related to the fact that the elastic properties are mostly determined by the porous dense tissue-effect of the presence of biological fluids and soft tissue in the porous space plays only a minor role. On the other hand, namely these constituents are responsible for bone's electrical conductivity. Cross-property connection for materials of this kind has been derived by Berryman and Milton (1988) in the form of inequality for the case when both the constituents as well as the overall material are isotropic.

In the text to follow, we express the tensor of elastic compliances in terms of the overall conductivity tensor in the closed form using the similarity between the microstructural parameters governing the two said properties. The results are validated experimentally and significant correlation between experimental and theoretical results has been observed.

2. Modeling of the microstructure of cortical bone

Overall electrical and elastic properties of cortical bone are largely determined by their microstructure comprising a large number of interconnected diverse pores filled with electrically conductive biological liquids and soft tissue-blood, lymph, nerve tissue etc.

In our analysis, based on description given by Martin and Burr (1989), Currey (2002) and Fung (1993) sketched in Fig. 1, we model cortical bone as a porous elastically transversely isotropic material of low electrical conductivity comprised of three systems of pores filled with elastically soft and electrically highly conductive tissue. Haversian canals are modeled as a system of parallel cylindrical pores (strongly prolate spheroidal inhomogeneities, Mura, 1987), in which their axes of geometrical symmetry coincide with the axes of the material symmetry of the matrix (the axis of transverse-isotropy, x_3). The osteocyte lacunae, modeled as oblate spheroidal cavities in a plane of transverse-isotropy (planes normal to Haversian canals). Canaliculi and Volkman's canals are treated as a set of thin cylindrical pores, with the axes of rotation perpendicular to the axis of transverse-isotropy of the matrix, which means they are lying in planes of transverse-isotropy and are randomly oriented in these planes.

* Corresponding author.
E-mail address: igor@nmsu.edu (I. Sevostianov).

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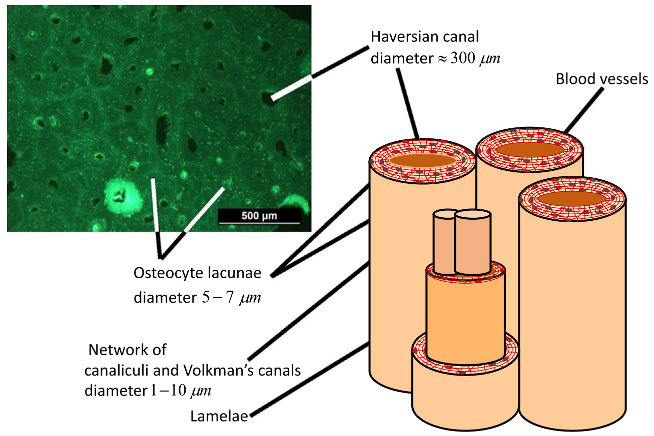

Fig. 1. Microstructure of cortical bone used in the present model: it is formed by osteons surrounding Haversian canals that contain blood and lymph vessels and nerves. Volkman's canals and canaliculi are randomly oriented in the planes orthogonal to the Haversian canals. The lamellae in osteons contain osteocytes located in oblate spheroidal pores (lacunae).

The aspect ratios of the spheroids have been estimated as follows

- For Haversian canals we calculated the aspect ratio as $\gamma = 3h/4R$, where h is the height of the osteon (we used average value of 4 mm) and R is the radius of the Haversian canal (we used 25 μm). Thus calculated aspect ratio $\gamma = 120$ allows one to preserve the volume of the inhomogeneity with fixed radius.
- Similarly, for Volkman's canals and canaliculi (accounted together) we used for h the difference between the radius of the osteon and the radius of the Haversian canal (125 μm) and for R, we used average radius of the canals (1.5 μm). The calculated aspect ratio is 80.
- The aspect ratio of the lacunae was taken as 0.2 in accordance with the observations of Currey (1962, 2002).

Although pores of different types have very different sizes, their partial porosities are comparable. Indeed, 1 mm^3 of the bone typically contains about 25,000 of osteocyte lacunae with the total surface area 5 mm^2/mm^3, about 106 canaliculi with the total surface area of 160 mm^2/mm^3 and about 20 Haversian canals with the total surface area of 3 mm^2/mm^3 (Martin and Burr, 1989). These numbers imply partial porosities for each of these types in the range 0.075–0.120. Thus, it does not seem adequate to attribute entire porosity of the cortical bone to the Haversian canals as in the model proposed by Dong and Guo (2004, 2006).

As stated above, the channels and pores contain blood and lymph vessels, nerve fibers, and living cells. The influence of these fluids and soft tissues can be neglected in the context of the overall *elastic* response. Indeed, elastic stiffness of the mineralized tissue is of the order of several *GPa*, while Young's moduli of blood vessels are of the order of 10 MPa at pressures of 100 mm Hg, (Wesly et al., 1975). Young's moduli for the nerve tissue and for the cells are of the order of 4 – 10 MPa (Beel et al., 1984), and of 1 kPa, (Theret et al., 1988), correspondingly. Thus, in the context of elastic properties, we treat the pores as empty ones embedded in the dense tissue that represents a combination of collagen fibers (protein), and hydroxyapatite $Ca_{10}(PO_4)_6(OH)_2$ crystals (mineral), (Katz, 1980). Mineralized tissue possesses transversally isotropic mechanical properties (Currey and Zioupos, 2001); in our calculations, we used the extrapolated data of Dong and Guo (2006) for dense tissue (Table 1) as well as our own measurements. In the context of electrical properties, we model the mineralized tissue as the isotropic background of very low conductivity, thus ignoring the bone matrix anisotropy since electrical conductivity of the matrix being different in different directions is still very small. The electrical conductivity of the bone matrix was extrapolated from the measurements to zero porosity and taken for calculation of cross-property coefficients as $k_0 = 3.841$ mS/m (see Section 6). The effect of the conductive soft tissue on the overall electrical properties of the bone is dominant; for the conductivity of the soft tissue we used $k_1 = 1.5$ S/m according to Hirsh et al. (1950), Visser (1992), and Hoetink et al. (2004).

Table 1
The transversely isotropic elastic constants of cortical bone calculated based on mechanical testing done by Dong and Guo (2004).

E_1(GPa)	E_3(GPa)	ν_{31}	G_{12}(GPa)	G_{13}(GPa)
11.419	24.16	0.38	3.9	6.54

3. Background material: property contribution tensors

We consider a homogeneous elastic material (matrix), with the compliance tensor $\boldsymbol{S}^0$ and electrical resistivity $\boldsymbol{r}^0$ containing an inhomogeneity of volume V_1 with the compliance tensor $\boldsymbol{S}^1$ and electrical resistivity $\boldsymbol{r}^1$. Compliance contribution tensors have been first introduced in the context of ellipsoidal pores and cracks in isotropic material by Horii and Nemat-Nesser (1983). For general case of ellipsoidal elastic inhomogeneities embedded in an isotropic matrix, these tensors were formally defined and calculated by Sevostianov and Kachanov (1999, 2002). Sevostianov et al. (2005) calculated components of this tensor for a spheroidal inhomogeneity embedded in a *transversely-isotropic* material. The compliance contribution tensor of the inhomogeneity is a fourth-rank tensor $\boldsymbol{H}$ that gives the extra strain (per reference volume V) due to its presence:

$$\Delta \boldsymbol{\epsilon} = \frac{V_1}{V}\boldsymbol{H} : \boldsymbol{\sigma}^{\infty}, \quad or, \text{ in components, } \Delta\epsilon_{ij} = \frac{V_1}{V}H_{ijkl}\sigma_{kl}^{\infty} \tag{3.1}$$

where σ_{kl}^{∞} are remotely applied stresses that are assumed to be uniform within V in the absence of the inhomogeneity.

In the case of multiple inhomogeneities, the effective compliance, calculated in the non-interaction approximation (NIA) is given by

$$S_{ijkl} = S_{ijkl}^0 + \frac{1}{V}\sum_m V_m H_{ijkl}^{(m)} \tag{3.2}$$

For an *ellipsoidal* inhomogeneity, its compliance contribution tensor is expressed in terms of Hill's tensor P_{ijkl} (Hill, 1963, Walpole, 1969) as

$$\boldsymbol{H} = \left[\left(\boldsymbol{S}^1 - \boldsymbol{S}^0\right)^{-1} + \boldsymbol{C}^0 : \left(\boldsymbol{J} - \boldsymbol{P} : \boldsymbol{C}^0\right)\right]^{-1}, \tag{3.3}$$

Hence, the problem of calculating the components of H_{ijkl} for an ellipsoidal inhomogeneity is reduced to the calculation of Hill's tensor. The expressions for the components P_{ijkl} of this tensor for a spheroidal inhomogeneity aligned with the axis of a transversely-isotropic material were derived by Sevostianov et al. (2005) in the following form

$$P_{ijkl} = p_1 T_{ijkl}^1 + p_2 T_{ijkl}^2 + p_3 T_{ijkl}^3 + p_4 T_{ijkl}^4 + p_5 T_{ijkl}^5 + p_6 T_{ijkl}^6 \tag{3.4}$$

where basic tensors T_{ijkl}^m and coefficients p_1 are given in the Appendix. Using formula (3.3) and the rule of multiplication for tensors represented in terms of standard tensor basis allow one to calculate components of the compliance contribution tensor. Fig. 2a and b illustrates dependence of the components of tensors