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A bioenergetic model for simulating athletic performance of intermediate duration

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A R T I C L E I N F O

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A B S T R A C T

Simulating factors affecting human athletic performance, including fatigue, requires a dynamic model of the bioenergetic capabilities of the athlete. To address general cases, the model needs inputs, outputs, and states with a set of differential equations describing how the inputs affect the states and outputs as functions of time. We improve an existing phenomenological muscle model, removing unnecessarily fast dynamic behavior, adding force–velocity dependence, and generalizing it to task level activities. This makes it more suitable for simulating and calculating optimal strategies of athletic events of medium duration (longer than a sprint but shorter than a marathon). To examine the validity and limitations of the model, parameters have been identified from numerical fits to published experimental data.

1. Introduction

When considering athletic events that involve racing against the clock such as a 5 km road race or cycling time trial, the following question inevitably arises: what is the optimal pacing strategy? Commonly practiced strategies range from maintaining a constant power output to finishing with a sprint. The question is especially difficult to answer in situations where the athlete's speed and power output vary over the duration of the event (e.g., a course with hills or wind). The first step in answering it consists of determining a model to represent the athlete's energetic dynamics in the event.

Numerous authors have examined competitive athletic pacing. A runner's optimal strategy for different distances was calculated and the results were compared to the then current world records (Keller, 1974). This model used the concept of an oxygen debt. Anaerobic metabolism effects on energy production have also been investigated (Ward-Smith, 1999), but in a manner which is not applicable to longer events or events with a variable power output. The minimum time problem for running has been studied using the critical power model by Morton (2009) who found that the optimal solution is all-out for the entire race. All these efforts have limitations, either in fidelity or extensibility to more complex events.

Simulating an athlete's performance in a general situation requires a dynamic model with a control input which governs the athlete's output. While such dynamics models have existed for decades, a more recent approach (Liu et al., 2002) divides the muscle into three compartments: resting, active, and fatigued. This model has also been extended (Xia and Frey Law, 2008) to include multiple muscle fiber types and the control logic required to activate the fibers in a realistic fashion. More recently, a 4 compartment, single fiber type model was proposed which appears to faithfully replicate experimental behavior (Sih et al., 2012).

However, if the goal is to perform optimal control calculations which can tell an athlete what power output, as a function of time, they should produce to achieve the minimum elapsed time in a cycling time trial, none of these models are appropriate. The models above all have significant weaknesses which prevent them from filling this role. To best answer the question, "what is the optimal strategy?", we present a new phenomenological model which represents as much behavior as needed and no more.

In this paper, we present a new dynamic model, designed for answering pacing strategy questions. It is an improvement of the existing phenomenological model of Xia and Frey Law (2008); it is more appropriate for general simulation and use in optimal control calculations. It does not attempt to replace current knowledge on muscle physiology, nervous system control, or decades of clinical experiments. Instead, it attempts to represent what an athlete can do, without considering the details of the physiological mechanisms of how this happens. Compared to current models, it removes unnecessary detail and adds important features. A limited validation using published experimental data is shown. Finally, interpretations of the identified physiological parameters are discussed.

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2. Methods

A qualitative overview of the model is first presented, followed by a mathematical description, and the methods used for its validation.

2.1. Background

Compartment theory has been used for modeling a single muscle (Liu et al., 2002; Xia and Frey Law, 2008). Each motor unit (MU) pool within the muscle is a collection of a finite number of similar fibers, but for modeling purposes it is convenient to treat it instead as continuous. The continuous pool is apportioned into three compartments: resting fraction M_r, active fraction M_a, and fatigued fraction M_f. The dynamics of the model describe the transfer between compartments: active muscle becoming fatigued, fatigued muscle recovering, and resting muscle becoming active. Fig. 1 shows a schematic of this process during and after a maximal effort exercise.

We have taken the approach of Xia and Frey Law (2008) a step further by applying it to groups of muscles, and use a 3 compartment model with multiple MU pools to represent task-level activities such as cycling, as the model of Sih et al. (2012) did with success. The fundamentals of the 3 compartment model are unchanged: the amount of force produced is proportional to the amount of active muscle, the rate at which fatigue accrues is proportional to how much muscle is active, and the rate of recovery is proportional to the current level of fatigue. There are differences in the model we present however, in the rate of activation, elimination of redundant quantities, and the inclusion of the force–velocity curve, among other changes. Additionally, the original model was developed for examining the behavior of motor unit pools within a single muscle. The present model was formulated with the assumption it could also represent a group of muscles. Fig. 2 shows how the compartments are allocated within a single motor unit pool.

2.2. Model dynamics

Unlike the Xia and Frey Law (2008) model, we assume there are only slow and fast fibers (but no intermediate ones). The quantities involved with the slow oxidative fiber MU pool are identified by a superscript o and the fast glycolytic fiber MU pool quantities by a superscript g. The differential equations for the single state variable for each pool, the fatigued muscle fraction M_f, are

$$\frac{dM_f^g}{dt} = M_a^g F^g - M_f^g R^g \tag{1a}$$

$$\frac{dM_f^o}{dt} = M_a^o F^o - M_f^o R^o \tag{1b}$$

where F^g, R^g and F^o, R^o are the fatigue and recovery rates for the fast and slow fiber types, respectively. As will be shown, only one differential equation per pool is required. This has been accomplished by removing mathematically redundant equations and instead partitioning the motor unit pools into fatigued and non-fatigued compartments (where it is assumed the non-fatigued compartment can be instantaneously apportioned between resting and active).

This model, which has 3 compartments per pool, can be represented with only one state variable for each MU pool for two reasons. First, there is clearly a redundant state, due to the total muscle size constraint

$$1 = M_a + M_f + M_r \tag{2}$$

making one state dependent on the other two.

Second, by not considering the *rate* of muscle activation, M_a no longer needs to be considered a state variable. The purpose of the present model is to simulate athletic events of medium duration (longer than a sprint but shorter than a marathon). We claim that the sub-second (much shorter than a second) time dynamics of muscle fiber recruitment are not relevant to athletic events of this and longer durations. If these sub-second dynamics are not considered, the non-fatigued muscle in a MU pool can be instantaneously split between the active and resting compartments. This allows for the control input to the system to simply be the *amount* of active muscle M_a, rather than the *recruitment rate* of active muscle.

These new control inputs (M_a for each fiber type) have constraints. Within each pool the amount of active muscle must be non-negative, and it cannot be greater than the amount of non-fatigued (i.e. available for recruitment) muscle

$$0 \le M_a^g \le 1 - M_f^g \tag{3a}$$

$$0 \le M_a^o \le 1 - M_f^o \tag{3b}$$

There is also a constraint imposed by the prioritization of fiber recruitment. Previous compartment models have used a logic sequence to enforce the correct order of fiber recruitment (Henneman et al., 1965). The following constraint equation enforces the same behavior:

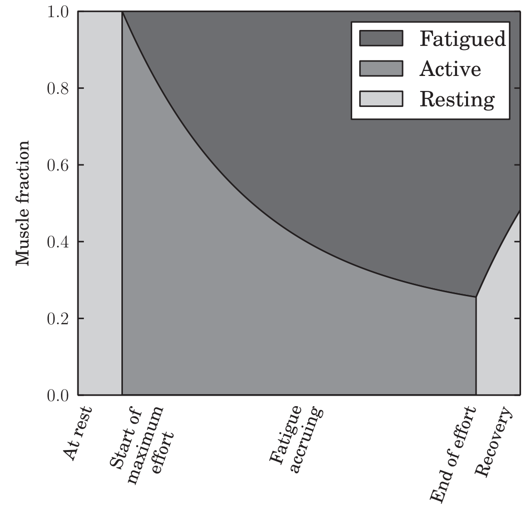

Fig. 1. A schematic representation of muscle transfer between compartments before, during, and after a maximal effort exercise. Initially the muscle is completely at rest. Once maximum effort starts, no muscle is at rest, and fatigue builds up. At the end of the effort period, recovery begins with active muscle being released and fatigued muscle recovering.

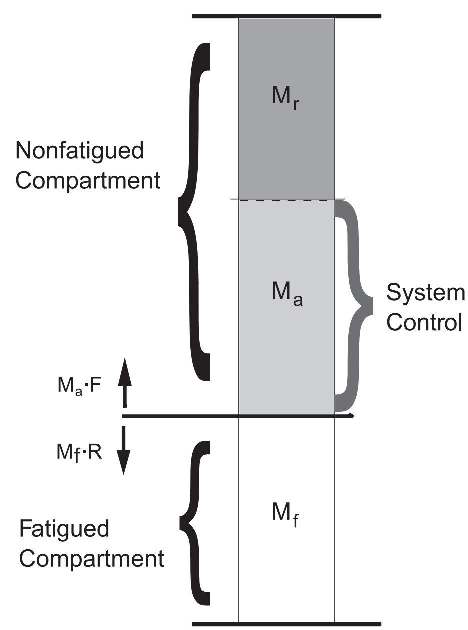

Fig. 2. A visualization of the compartments within a MU pool. The bar dividing the fatigued and non-fatigued compartments moves down at a rate which is the difference of the recovery ($M_f R$) and fatigue ($M_a F$). The system control M_a is bounded between zero and the amount of non-fatigued muscle. The resting compartment M_r is simply the non-active, non-fatigued fraction.

$$M_a^g \left(M_a^o + M_f^o - 1 \right) = 0 \tag{4}$$

The allowable space for the control inputs to the system is shown as a bold line in Fig. 3.

This constraint enforces that the amount of active fast fiber can be nonzero only when all of the slow fibers are active. Using a constraint equation is necessary when applying optimal control techniques, because logic sequences introduce non-smoothness, possibly preventing an optimal solution from being found. Outside optimal control applications, an equivalent logic sequence is acceptable.

The final change from other compartment models is the incorporation of the force–velocity curve within the equations. For medium duration events an athlete is unlikely to spend time at either extreme of the force–velocity curve, and we claim that an exponential force–velocity approximation is sufficient. This allows the