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An analytical model to predict interstitial lubrication of cartilage in migrating contact areas

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ABSTRACT

For nearly a century, articular cartilage has been known for its exceptional tribological properties. For nearly as long, there have been research efforts to elucidate the responsible mechanisms for application toward biomimetic bearing applications. It is now widely accepted that interstitial fluid pressurization is the primary mechanism responsible for the unusual lubrication and load bearing properties of cartilage. Although the biomechanics community has developed elegant mathematical theories describing the coupling of solid and fluid (biphasic) mechanics and its role in interstitial lubrication, quantitative gaps in our understanding of cartilage tribology have inhibited our ability to predict how tribological conditions and material properties impact tissue function. This paper presents an analytical model of the interstitial lubrication of biphasic materials under migrating contact conditions. Although finite element and other numerical models of cartilage mechanics exist, they typically neglect the important role of the collagen network and are limited to a specific set of input conditions, which limits general applicability. The simplified approach taken in this work aims to capture the broader underlying physics as a starting point for further model development. In agreement with existing literature, the model indicates that a large Peclet number, *Pe*, is necessary for effective interstitial lubrication. It also predicts that the tensile modulus must be large relative to the compressive modulus. This explains why hydrogels and other biphasic materials do not provide significant interstitial pressure under high *Pe* conditions. The model quantitatively agrees with *in-situ* measurements of interstitial load support and the results have interesting implications for tissue engineering and osteoarthritis problems. This paper suggests that a low tensile modulus (from chondromalacia or local collagen rupture after impact, for example) may disrupt interstitial pressurization, increase shear stresses, and activate a condition of progressive surface damage as a potential precursor of osteoarthritis.

1. Introduction

McCutchen slid cartilage against a large glass flat and was the first to propose interstitial (weeping) lubrication to explain the unusual response (McCutchen, 1959). He noted that fluid pressure, which develops under loading, reduced friction by $\sim 10\text{–}100\times$ while boundary lubrication with synovial fluid reduced friction by $\sim 2\times$. In the joint, interstitial pressurization increases load capacity (Ateshian et al., 1994), shields the matrix from stresses (Mow and Lai, 1980), signals the biochemical response (Wong et al., 2003; Carter et al., 2004), and reduces friction and wear (McCutchen, 1962; Soltz and Ateshian, 2000; Ateshian, 2009).

Generally speaking, interstitial fluid pressure subsides over time. Direct measurements of the fluid load fraction have shown that friction obeys the following relationship with the equilibrium friction coefficient, μ_{eq}, and the time-dependent fluid load fraction, F' (Krishnan et al., 2004):

$$\mu = \mu_{eq}(1 - F') \tag{1}$$

McCutchen recognized that interstitial lubrication must be restored *in-vivo* and proposed that dynamic loading and unloading was responsible (McCutchen, 1962). However, this hypothesis was rejected by direct observations of time-dependent friction during dynamic loading (Krishnan et al., 2005). In 2008, it was discovered that interstitial lubrication is maintained during sliding when cartilage is self-mated (Caligaris and Ateshian, 2008). In a follow-up test, the authors demonstrated that a rigid impermeable sphere, when slid against cartilage, also maintained low friction. They proposed that fluid pressure is maintained when hydrated tissue is continually introduced into the contact; they call this the migrating contact area (**MCA**). This discovery explained how fluid pressure is maintained *in-vivo*. Based on prior modeling of

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biphasic cylindrical layers in rolling contact (Ateshian and Wang, 1995), Ateshian proposed that fluid load support is sustainable in MCA when the Peclet number (Pe)$\gg$1 and negligible when $Pe \leq 1$; $Pe = Va/(H_a k)$ where V is sliding speed, a is the contact radius, H_a is aggregate modulus, and k is permeability (Ateshian, 2009).

Despite the rapid recent advancements in the field of cartilage lubrication, there remain major gaps that inhibit our ability to predict how tribological conditions and material properties impact tissue function. The state of the art provides a relationship between friction and fluid load fraction (Krishnan et al., 2004; Ateshian, 2009), but there remains no analytical expression to quantitatively relate the Peclet number to the fluid load fraction for MCA sliding conditions. This paper describes and experimentally supports an analytical model that relates measureable material properties and controllable mechanical conditions to the fluid load fraction and related functional parameters, including contact radius, effective contact modulus, contact stress, fluid pressure, friction coefficient, and shear stress.

2. Model

The force response of cartilage to deformation consists of components due to elastic stresses and those due to fluid pressure (McCutchen, 1962; Mow et al., 1980). The coupling of elastic deformation and fluid flow creates a challenging non-linear contact mechanics problem. To improve the tractability of the Hertzian contact problem, we initially treat the solid and fluid mechanics independently. Although cartilage violates nearly each of Hertz's assumptions, we find that Hertz's theory provides a reasonable contact model when the contact diameter is less than the cartilage thickness. The elastic foundation model is more appropriate in physiological conditions and the analysis follows an identical strategy.

We develop the Hertz solution over the elastic foundation solution here because we can test the Hertz solution under controllable experimental conditions. Consider the indentation of a rigid impermeable sphere into cartilage as illustrated in Fig. 1. According to Hertz's theory, the elastic force component, F_e, is the following function of sphere radius, R, contact modulus (a material property), $E_{c0}=E/(1-\nu^2)$, and penetration depth, $\delta=a^2/R$:

$$F_e = \frac{4}{3}\frac{E}{1-\nu^2}R^{0.5}\delta^{1.5} = \frac{4}{3}E_{c0}R^{0.5}\delta^{1.5} = \frac{4}{3}\frac{E_{c0}a^3}{R} \tag{2}$$

Volume-changing deformations like indentation cause interstitial fluid flow. According to Darcy's law, fluid flow through a permeable medium induces a pressure gradient:

$$\frac{dP}{dx} = \frac{V}{k} \tag{3}$$

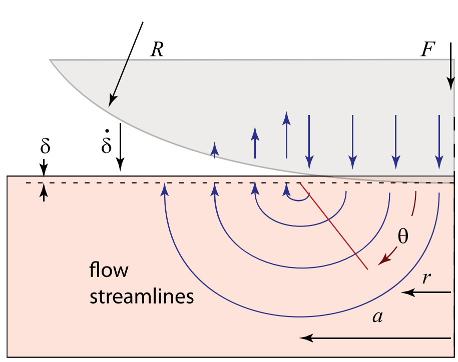

Fig. 1. (a) Axisymmetric contact model of a rigid sphere indenting cartilage. Streamlines show likely paths of fluid flow.

where V is the flow speed along a streamline, k is the permeability of the solid to the fluid of interest, and dP/dx is the pressure gradient along the streamline. Finite element models (Pawaskar et al., 2010; Accardi et al., 2011) have demonstrated that the streamlines during indentation and sliding approximate semi-circular arcs as shown in Fig. 1. Each streamline starts at the sphere surface at a distance r from the axis of symmetry with a speed of $\dot{\delta}$ in the compression direction. Conserving volume along each streamline gives the velocity as a function of starting point, r, and angle, θ:

$$V(r,\ \theta) = \frac{\dot{\delta}r}{a-(a-r)\cos(\theta)} \tag{4}$$

Assuming that the pressure outside the tissue is zero, Darcy's law can be integrated along each streamline to obtain the pressure acting on the sphere as a function r. The matrix compacts downward at a rate $\dot{\delta}$ under the contact so there is no relative flow at $\theta=0$. We estimate the relative flow rate by considering only the transverse component of V within the contact (*i.e.* when $0<\theta<\pi/2$). In this case, pressure on the counterbody takes the form:

$$P(r) = \int_0^{\pi/2} \frac{\dot{\delta}r(a-r)\sin(\theta)}{k(a-(a-r)\cos(\theta))}d\theta + \int_{\pi/2}^{\pi} \frac{\dot{\delta}r(a-r)}{k(a-(a-r)\cos(\theta))}d\theta \tag{5}$$

Integrating the pressure distribution yields an estimate of the fluid pressure force contribution[1]:

$$F_p = 1.37\frac{\dot{\delta}a^3}{k} \cong \frac{4}{3}\frac{\dot{\delta}a^3}{k} \tag{6}$$

The fluid load fraction, F', is the primary metric of interstitial lubrication. By definition, F' is the ratio of the fluid pressure force contribution and the total applied normal force. Inserting Eqs. (2) and (6) into this definition yields:

$$F' = \frac{F_p}{F_p+F_e} = \frac{\frac{4}{3}\frac{\dot{\delta}a^3}{k}}{\frac{4}{3}\frac{\dot{\delta}a^3}{k}+\frac{4}{3}\frac{E_{c0}a^3}{R}} = \frac{Pe}{Pe+1} \tag{7}$$

where $Pe \equiv \dot{\delta}R/E_{c0}k$ for indentation.

The mechanics of a migrating contact are analogous to those of indentation. When the sphere in Fig. 1 travels a distance, a, the tissue is consolidated by δ. Thus, the average deformation rate is: $\dot{\delta} = V\delta/a$. By definition from Hertz theory, $\delta = a^2/R$, so the Peclet number for sliding becomes: $Pe \equiv Va/E_{c0}k$, which is identical to that reported previously (Ateshian, 2009).

Eq. (7) suggests that $F' \to 1$ as $\dot{\delta} \to \infty$ and only holds for an infinite tensile modulus. For real materials, Eq. (7) is limited to an asymptotic limit that depends on the elastic properties and contact geometry. Soltz and Ateshian demonstrated that this asymptotic limit for F' in unconfined compression is essentially governed by the ratio of tensile modulus to compressive modulus, E^* (Soltz and Ateshian, 2000). The same mechanism applies here and is important to understand. Consider Fig. 2, which illustrates the unconfined compression of a biphasic material. Assume that Poisson's ratio is 0 (Soltz and Ateshian, 2000, show that this is true to an excellent approximation) and that the deformation shown occurs instantaneously (flow cannot occur). The deformed shape conserves volume and the transverse strains are half the normal strain. Soltz and Ateshian use E_{-Y} and E_{+Y} to represent the compressive and tensile moduli, respectively. The transverse tensile stress on the matrix is: $\sigma_+ = (\sigma_-/2)(E_{+Y}/E_{-Y}) = (\sigma_-/2)E^*$. If the interface is frictionless, fluid pressure, P, must balance the tensile stress (globally speaking) and $\sigma_- = 2P/E^*$. Therefore, the fluid load fraction is $F' = P/(P+\sigma_-) =$

[1] The approximate constant of 4/3 was misrepresented as 2/3 in our previous paper (Bonnevie et al., 2012).