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Short communication

Optimal average path of the instantaneous helical axis in planar motions with one functional degree of freedom

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ABSTRACT

This paper presents a model for determining the path of the instantaneous helical axis (IHA) that optimally represents human planar motions with one *functional* degree of freedom (fDOF). A human movement is said to have one fDOF when all degrees of freedom (DOFs) are coordinated such that all the kinematic variables can be expressed, across movement repetitions, as functions of only one independent DOF, except for a small natural intercycle variability quantified as lower than a prespecified value. The concept of fDOF allows taking into account that, due to motor coordination, human movements are executed in a repeatable manner. Our method uses the measurement of several repetitions of a given movement to obtain the optimal average IHA path. The starting point is a change of variables, from time to a joint position magnitude (generally an angle). In this way, instead of operating with the time-dependent single-valued trajectory of the successive cycles, our model permits the representation of any motion variable (e.g. positions and their time derivatives) as a cloud of points dependent on the joint angle. This allows the averaging to be performed over the displacements and their derivatives before determining the mean IHA path. We thus avoid the nonlinear magnification of errors and variability inherent in the IHA computation. Moreover, the IHA path can be considered as a geometric attribute of the joint and the type of motion, rather than of each single movement execution. An experiment was performed that show the accuracy and usefulness of the method.

1. Introduction

The instantaneous helical axis (IHA) has been used to describe the kinematics of complex joints such as the neck (Woltring et al., 1994), the lumbar spine (Page et al., 2009b), the knee (Wolf and Degani, 2007), the shoulder complex (Dempster, 1955) or the ankle (Leardini et al., 1999).

Most studies based on the IHA operate on the six degrees of freedom (DOFs) that describe the full possibilities of movement. However, the DOFs of a complex joint do not vary independently of each other during the execution of a natural motion. Due to motor coordination, each person executes a type of movement in a repeatable way and there are relations between the kinematic variables. Then, the number of *independent* variables needed to represent the movement is lower than six. These independent DOFs are called *functional* degrees of freedom (fDOFs) (Li, 2006). Many movements used in clinical examinations of joint function are actually very simple and often planar. In some cases, most of the motion variability can be explained by only one joint variable (Page et al., 2008), and we can assume a single fDOF.

The existence of a single fDOF does not ensure that a single path of the IHA will be obtained when several repetitions of the same movement are performed. Errors, artifacts and natural intrasubject variability introduce some dispersion in the kinematic variables, which is strongly and nonlinearly magnified when calculating the IHA (Woltring et al., 1994). This leads to erratic results and negatively affects the estimation of an average representation of movement (Page et al., 2006b).

In this paper, we propose a robust model to determine the average path traced by the IHA in planar movements with one fDOF. Instead of computing the IHA from time-dependent kinematic variables, we represent the IHA path as a function of the joint angle variable. This way, repetitions of the same motion can be efficiently averaged across cycles before determining the IHA, thus reducing the dispersion of results that appears when working with instantaneous variables. The model and its effectiveness are illustrated with an experiment.

2. Kinematic model and averaging process

2.1. Kinematic model

The experimental determination of the IHA associated with joint motion involves analyzing both the position and velocity

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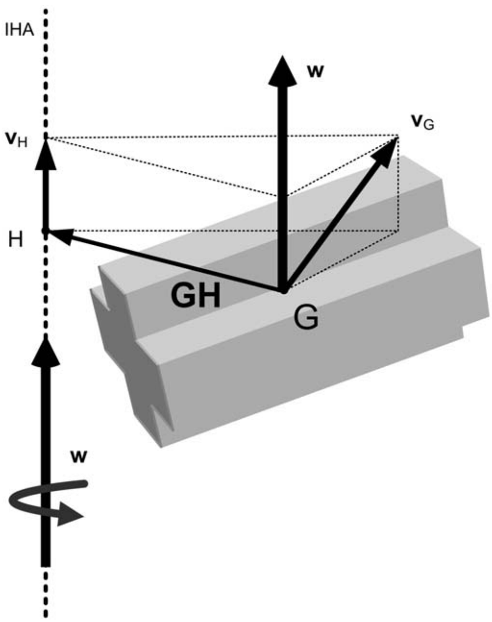

Fig. 1. Variables involved in the calculation of the instantaneous helical axis (IHA). Consider the body depicted in the figure as a distal human segment. Then, all the represented variables refer to the motion of this distal segment relative to a proximal one, which is not shown. Point H is the point of the IHA closest to point G.

variables of the distal segment relative to the proximal one. The position analysis provides the coordinates of a point G, $\mathbf{R}_G(t)$, and a measure of the orientation, e.g. the attitude vector, $\boldsymbol{\theta}(t)$ (Woltring et al., 1994). The velocity analysis provides the velocity of G, $\mathbf{v}_G(t)$, and the angular velocity, $\mathbf{w}(t)$. The IHA is the line parallel to $\mathbf{w}$ that passes through point H given by:

$$\mathbf{GH}(t) = \mathbf{w}(t) \times \mathbf{v}_G(t)/w^2(t) \tag{1}$$

(see Fig. 1 for an illustration). If the motion is planar, then we can expect up to three DOFs. If the condition of having one fDOF is satisfied, $\mathbf{R}_G$ may be represented as a function of the joint angle θ. In addition, $\mathbf{v}_G$ may be expressed as a function of the joint angle and its derivative (the angular velocity, w):

$$\mathbf{R}_G = \mathbf{R}_G(\theta) \tag{2}$$

$$\mathbf{w} = \frac{d\theta}{dt} \quad \mathbf{u} = w\mathbf{u} \tag{3}$$

$$\mathbf{v}_G = \frac{d\mathbf{R}_G}{d\theta}\frac{d\theta}{dt} = \mathbf{v}_S(\theta)w \tag{4}$$

where $\mathbf{u}$ is the unit vector perpendicular to the plane of motion and $\mathbf{v}_S$ is a standardized velocity representing the displacement of G per unit of joint rotation. Note that $\mathbf{v}_S$ does not depend on time or on speed of motion. Substituting (3) and (4) into (1) yields the following expression for the location of the IHA:

$$\mathbf{GH}(\theta) = \mathbf{u} \times \mathbf{v}_S(\theta) = \mathbf{u} \times d\mathbf{R}_G(\theta)/d\theta \tag{5}$$

Eq. (5) shows that by performing a change of variables, moving from the time domain to the angle domain, the IHA can be viewed as a geometric attribute of the joint and type of movement that does not depend on the speed of motion. This eliminates a source of errors, namely the term $1/w^2$ from Eq. (1).

2.2. Averaging process

In practice, the relation between $\mathbf{R}_G$ and θ is not completely single-valued, due to the small natural variability across repetitions of the same motion. This makes it necessary to perform some averaging across cycles, with the aim of obtaining a mean path of the IHA that provides the best possible representation of the average movement.

Since the relation between the IHA location and the motion variables is nonlinear, the direct averaging of the different individual IHA paths does not ensure average values of the positions and velocities when the motion is reproduced using such averaged path. To obtain an IHA path that optimally represents the mean movement, it is better to first average both position and velocity variables ($\mathbf{R}_G$ and $\mathbf{v}_G$); these averaged values will then give us the optimal average IHA path. Data fitting is required to obtain a mean across cycles of $\mathbf{R}_G(\theta)$, denoted by $\langle\mathbf{R}_G(\theta)\rangle$, whose derivative is the average across cycles of $\mathbf{v}_S(\theta)$, denoted by $\langle\mathbf{v}_S(\theta)\rangle$. This problem involves an optimization procedure with two fitting criteria, and can be suitably solved using the technique of smoothing with regularization (Ramsay and Silverman, 2005). The regularization consists of smoothing the function $\mathbf{R}_G(\theta)$ while imposing some conditions on its derivative $d\mathbf{R}_G/d\theta$. The calculation procedure can be summarized in the following steps:

(1) Perform a 3D kinematic analysis from measurements of a cyclic planar motion to obtain the position variables ($\mathbf{R}_G(t)$,$\boldsymbol{\theta}(t)$) and velocities ($\mathbf{v}_G(t)$,$\mathbf{w}(t)$) (Page et al., 2009a). To obtain a robust estimation of helical variables it is advisable to record several cycles (three or more).
(2) Determine the unit vector perpendicular to the plane of motion, $\mathbf{u}$, by averaging $\mathbf{w}(t)$. The averaging must be computed separately for back and forth movements otherwise the average value would vanish. At this point, check the condition of planar motion by verifying the following constraints: first, the angle between $\mathbf{u}$ and $\mathbf{w}(t)$ must be small; second, the linear and angular velocities must be sufficiently perpendicular. We denote as $\theta = \boldsymbol{\theta} \cdot \mathbf{u}$ the joint variable associated with the planar movement.
(3) Check the condition of one fDOF by applying the procedure described in Page et al. (2008).
(4) Sort the values of θ in ascending order and arrange $\mathbf{R}_G$ and $\mathbf{v}_G$ according to θ, thus obtaining $\mathbf{R}_G(\theta)$ and $\mathbf{v}_G(\theta)$.
(5) Obtain the smooth average across cycles of $\mathbf{R}_G(\theta)$, denoted by $\langle\mathbf{R}_G(\theta)\rangle$, and its associated derivative, $\langle\mathbf{v}_S(\theta)\rangle$, following the double criterion of adjusting position and velocity variables. This can be accomplished with the procedure based on local polynomial regression described in Page et al. (2006a). The level of smoothing is given by the bandwidth parameter h (Ramsay and Silverman, 2005). Small values of h will provide high roughness in the estimation of $\langle\mathbf{v}_S\rangle$ and therefore large differences between the measured values of $\mathbf{v}_G(\theta)$ and those estimated from $\langle\mathbf{v}_S(\theta)\rangle$ by Eq. (4). On the contrary, if h is too high, then there is a bias in both $\langle\mathbf{R}_G(\theta)\rangle$ and $\langle\mathbf{v}_G(\theta)\rangle$. Thus the data fitting procedure with regularization can be done by smoothing $\mathbf{R}_G(\theta)$ with the bandwidth h that minimizes the least square error criterion:

$$\mathrm{LSE}(h) = \sum\left[\mathbf{v}_G(\theta) - \frac{d\theta}{dt}\langle\mathbf{v}_S(h,\theta)\rangle\right]^2 \tag{6}$$

(6) Once optimal values of $\langle\mathbf{R}_G(\theta)\rangle$ and $\langle\mathbf{v}_G(\theta)\rangle$ have been computed, determine, for each value of θ, the IHA as the line parallel to $\mathbf{u}$ that passes through point $H(\theta)$ given by Eq. (5). The path of the IHA calculated in this manner corresponds to the movement averaged across repetitions.

In the rest of the paper, we will refer to this way of determining the average path of the IHA as the "geometric approach", while the calculation of the IHA using Eq. (1) will be called the "instantaneous approach". Note that we have assumed that the