Journal of Biomechanics 43 (2010) 455–462

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Determination of wave speed and wave separation in the arteries using diameter and velocity

J. Feng [a,b], A.W. Khir [a,*]

[a] Brunel Institute for Bioengineering and School of Engineering and Design, Brunel University, Kingston Lane, Uxbridge, Middlesex UB8 3PH, UK
[b] School of Mechanical Engineering, Taiyuan University of Technology, Shanxi, PR China

ARTICLE INFO

Article history:
Accepted 29 September 2009

Keywords:
Noninvasive
Separated wave
Wave speed
Wave intensity analysis
Hemodynamic
Diameter
Velocity

ABSTRACT

The determination of arterial wave speed and the separation of the forward and backward waves have been established using simultaneous measurements of pressure (P) and velocity (U). In this work, we present a novel algorithm for the determination of local wave speed and the separation of waves using the simultaneous measurements of diameter (D) and U. The theoretical basis of this work is the solution of the 1D equations of flow in elastic tubes. A relationship between D and U is derived, from which, local wave speed can be determined; $C = \pm 0.5(dU_{\pm}/d\ln D_{\pm})$. When only unidirectional waves are present, this relationship describes a linear relationship between $\ln D$ and U. Therefore, constructing a lnDU-loop should result in a straight line in the early part of the cycle when it is most probable that waves are running in the forward direction. Using this knowledge of wave speed, it is also possible to derive a set of equations to separate the forward and backward waves from the measured D and U waveforms. Once the forward and backward waveforms of D and U are established, we can calculate the energy carried by the forward and backward waves, in a similar way to that of wave intensity analysis. In this paper, we test the new algorithm *in vitro* and present results from data measured in the carotid artery of human and the ascending aorta of canine. We conclude that the new technique can be reproduced *in vitro*, and in different vessels of different species, *in vivo*. The new algorithm is easy to use to determine wave speed and separate D and U waveforms into their forward and backward directions. Using this technique has the merits of utilising noninvasive measurements, which would be useful in the clinical setting.

1. Introduction

Several methods for the determination of local arterial wave speed, (c), have been proposed, and a number of these methods, used the simultaneous measurements of pressure and velocity at the same site. Westerhof et al. (1972) suggested the characteristic impedance can be used as an indication of c. Khir et al. (2001) introduced the *PU*-loop which was shown to have a linear portion in early systole and the slope of which also equals ρc, where ρ is blood density. Using the *PU*-loop method, Harada et al. (2002) introduced a noninvasive system for online measurement of c in patients. Davies et al. (2006) suggested a technique for the determination of local c, also using simultaneous measurements of pressure and velocity at the same site.

The separation of the forward and backward waves has been studied almost exclusively in the frequency domain for at least a quarter of a century (early seventies–mid nineties). More recently, an alternative technique was introduced by Parker and Jones (1990), which allows for the separation of waves as a function of time. Both of these wave separation techniques originally used invasive measurements of pressure and velocity.

Wave intensity analysis (WIA) was first presented by Parker et al. (1988). The method is based on the solution of the method of characteristics to the 1D equations of mass and momentum conservation using the simultaneous measurements of pressure and velocity. WIA also allows for the separation of intensities of the forward and backward directions (Parker and Jones, 1990). Invasive WIA has been used in hemodynamic studies throughout the arterial bed; the aorta (Koh et al., 1998), pulmonary circulation (Hollander, 2001), coronary arteries (Sun et al., 2000) and left and right ventricles (Lanoye et al., 2005; Sun et al., 2006). As WIA seemed to promise valuable clinical information, Niki et al. (1999) reported a noninvasive technique for measuring wave intensity (dI) in patients. Their technique is based on deducing the pressure waveform of the carotid artery by scaling its diameter waveform. Limits of the scale are the systolic and diastolic pressures of the brachial artery as measured with a cuff-type manometer.

To date, a full analysis of arterial wave propagation using direct measurements of velocity and diameter is lacking. Therefore, the

* Corresponding author. Tel.: +44 0 1895265857; fax: +44 0 1895274608.
E-mail address: ashraf.khir@brunel.ac.uk (A.W. Khir).

0021-9290/$ - see front matter
doi:10.1016/j.jbiomech.2009.09.046

aim of this work is to develop an algorithm that utilises the noninvasive, simultaneous measurements of diameter and flow velocity to (1) determine local wave speed, and (2) separate diameter, velocity and wave intensity waveforms into their forward and backward components.

2. Theoretical analysis

2.1. Determination of wave speed

Wave speed is a function of the distensibility of tube wall, D_s, and density of fluid, ρ

$$c^2 = \frac{1}{\rho D_s} \tag{1}$$

where D_s=(1/A) (dA/dP), dA is the change in a circular cross-sectional area and dP is the change in pressure. We can also write

$$\frac{dA}{A} = \frac{2dD}{D} \tag{2}$$

where dD is the change in diameter. Therefore, substituting Eq. (2) in (1) gives

$$dP = \rho c^2 \frac{2dD}{D} \tag{3}$$

We define the diameter elemental waves as the changes in diameter in response to the effect of the forward and backward running fluid waves. Defining the change in velocity as dU, a forward acceleration ($dP > 0$, $dU > 0$) or backward deceleration wave ($dP > 0$, $dU < 0$) will cause an increase in diameter ($dD > 0$). Oppositely, a forward deceleration ($dP < 0$, $dU < 0$) or backward acceleration wave ($dP < 0$, $dU > 0$) will cause a decrease in diameter ($dD < 0$).

Changes in pressure, dP, are considered as the linear summation of the changes in pressure in the forward (+) and backward (−) directions, $dP = dP_+ + dP_-$. It is also reasonable to consider changes in the vessel diameter, dD, as the linear summation of the changes in diameter in the forward and backward directions, $dD = dD_+ + dD_-$. Substituting for dP and dD into Eq. (3) gives

$$dP_+ + dP_- = \frac{2\rho c^2}{D}(dD_+ + dD_-) \tag{4}$$

Eq. (4) provides a relationship between dP and dD in (+) and (−), and the aim of the following steps is to replace the pressure with velocity.

The well-established water hammer equation was introduced by Kries in 1883 (Tijsseling and Anderson, 2007) can be written in the '+' and '−' directions as

$$dP_\pm = \pm \rho c dU_\pm \tag{5}$$

Substituting Eq. (5) into (4) gives

$$c = \frac{D}{2} \frac{(dU_+ + dU_-)}{(dD_+ + dD_-)} \tag{6}$$

If we consider $dD/D = d\ln D$, the incremental hoop strain, we can introduce the new expression of wave speed in terms of U and D

$$c = \pm \frac{1}{2} \frac{dU_\pm}{d\ln D_\pm} \tag{7}$$

Eq. (7) describes a linear relationship between U and $\ln D$ for unidirectional waves. Therefore, plotting $\ln D$ against U gives a $\ln DU$-loop, and we should expect a linear portion during the early part of systole, when it is most probable that reflected waves do not exist. In this work, we determined the linear part of the loop by fitting a straight line to the appropriate portion of the data by eye, as previously demonstrated in the PU-loop (Khir et al., 2001). The slope of the linear portion of the loop equals $\frac{1}{2}c$.

2.2. The separation of waves

Substitute the water hammer equation into Eq. (4) and divide by ρc gives

$$dU_+ + dU_- = c\frac{2(dD_+ + dD_-)}{D} \tag{8}$$

If only unidirectional waves are considered, Eq. (8) can be written as

$$dD_\pm = \pm \frac{D}{2c} dU_\pm \tag{9}$$

Eq. (9) is similar to the water hammer equation, but rather than relating the changes of pressure and velocity, it relates the changes of D and U. If we assume dU is additive in (+) and (−) directions, $dU = dU_+ + dU_-$, dD and dU in (+) and (−) directions can be derived

$$dD_\pm = \frac{1}{2}\left(dD \pm \frac{D}{2c} dU\right) \tag{10}$$

$$dU_\pm = \frac{1}{2}\left(dU \pm \frac{2c}{D} dD\right) \tag{11}$$

D and U waveforms in (+) and (−) directions can then be obtained by summing the instantaneous changes in D and U in each direction, respectively.

$$D_\pm = \sum_{t=0}^{T} dD_\pm + D_{(0)} \tag{12}$$

$$U_\pm = \sum_{t=0}^{T} dU_\pm + U_{(0)} \tag{13}$$

where T is the total cycle time. $D_{(0)}$ and $U_{(0)}$ are the integration constants and taken arbitrarily as zero.

Wave intensity can be calculated from the measured parameters as ${}_n dI = dDdU$, where ${}_n dI$ is the noninvasive wave intensity; we use the earlier nomenclature of wave intensity (dI), adding (n) to make a distinction between the invasive and noninvasive calculations. The separated ${}_n dI$ in the forward and backward directions is ${}_n dI_\pm = dD_\pm dU_\pm$, which can be written more explicitly using Eqs. (10) and (11) as

$${}_n dI_\pm = \pm \frac{1}{4(D/2c)}\left(dD \pm \frac{D}{2c} dU\right)^2 \tag{14}$$

Eq. (14) has all the useful qualities of traditional WIA. The equation is always positive in the forward and always negative in the backward direction, as will be seen in the results. Units of the traditional wave intensity are (W/m^2) while the units of Eq. (14) are those of diffusion (m^2/s). The difference between the physical meanings of the two approaches is not readily apparent and deserves further questioning.

3. Experimental methods

3.1. In vitro instrumentation

A pneumatically driven flexible diaphragm cardiac assist pump (BCM, Cardialcare Inc., St. Louis, USA) was used to generate a reproducible pulse in two flexible tubes (16 mm diameter with 1.8 m length; 24 mm diameter with 0.8 m length). Both tube sizes have wall thickness of 1 mm and Young's modulus of 198.7 kN/m^2. Each tube was mounted in a water-full tank to avoid undesired reflections. Both ends of each tube were connected to an overhead reservoir to provide an initial pressure in the system. Simultaneous measurements of pressure, flow-rate and diameter were taken at the same site, sequentially at intervals of