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Globographic visualisation of three dimensional joint angles

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ABSTRACT

Three different methods for describing three dimensional joint angles are commonly used in biomechanics. The joint coordinate system and Cardan/Euler angles are conceptually quite different but are known to represent the same underlying mathematics. More recently the globographic method has been suggested as an alternative and this has proved particularly attractive for the shoulder joint. All three methods can be implemented in a number of ways leading to a choice of angle definitions. Very recently Rab has demonstrated that the globographic method is equivalent to one implementation of the joint coordinate system.

This paper presents a rigorous analysis of the three different methods and proves their mathematical equivalence. The well known sequence dependence of Cardan/Euler is presented as equivalent to configuration dependence of the joint coordinate system and orientation dependence of globographic angles. The precise definition of different angle sets can be easily visualised using the globographic method using analogues of longitude, latitude and surface bearings with which most users will already be familiar. The method implicitly requires one axis of the moving segment to be identified as its principal axis and this can be extremely useful in helping define the most appropriate angle set to describe the orientation of any particular joint. Using this technique different angle sets are considered to be most appropriate for different joints and examples of this for the hip, knee, ankle, pelvis and axial skeleton are outlined.

1. Introduction

Joint angles define the orientation of a coordinate system (CS) representing one body segment relative to that describing another. In three dimensions there are three rotational degrees of freedom and three parameters are thus required. There is no obvious choice as to how best to select these parameters and various methods have been proposed.

Cardan/Euler[1] *angles* (Ayoub et al., 1974; Tupling and Pierrynowski, 1987) are used to describe three dimensional angles in most areas of mathematics and engineering. The angles are those through which a coordinate system must be rotated in sequence about orthogonal axes embedded within it to map it from that of the proximal body segment CS to that of the distal. The *Joint Coordinate System* (*JCS*—which is not really a coordinate system) was introduced by Chao (1980) and Grood and Suntay (1983) being based on experience with physical mechanisms linking segments on which goniometers had been mounted. In these the mechanism operated such that one hinge was aligned with an axis of the proximal CS, another with an axis of the distal CS and the third hinge was "floating"[2] so as to lie along the mutual perpendicular. Grood and Suntay in particular, generalised this approach suggesting that this method should be applied regardless of whether a physical mechanism were used or not. This approach has been promoted by the International Society of Biomechanics standard (Wu and Cavanagh, 1995; Wu et al., 2002, 2005).

The *globographic* method has received much less attention although Dempster (1956) comprehensively described its application to a range of joints as early as 1956 (see Fig. 1) and referred to it as "standard globographic representation" citing references from the German biomechanical literature as far back as 1865. It requires one axis of the moving segment to be considered the primary axis and describes its position by angles analogous to latitude and longitude and the rotation about this in a manner analogous to surface bearing. The technique has been rediscovered by a number of authors at various intervals since (MacConaill, 1956; Pearl et al., 1992a, 1992b, 1992c; Cheng, 2000, 2004; Cheng et al., 2000; Cheng and Pearcy, 2001; Doorenbosch et al., 2003) and given a range of names including *spherical rotation coordinate*

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[1] Use of these two eponymous terms (Cardan was a 16th Century Italian and Euler a 19th Century Swiss mathematician) is not consistent. Some make a distinction with Cardan angles assumed to be about three different axes and Euler angles using the same axis for the first and third rotations but it isn't at all clear how this originated and whether it is appropriate. *Cardan/Euler* will be used here to emphasise their equivalence.

[2] "Floating" is a mis-leading term as the axis is actually constrained by the condition of mutual perpendicularity.

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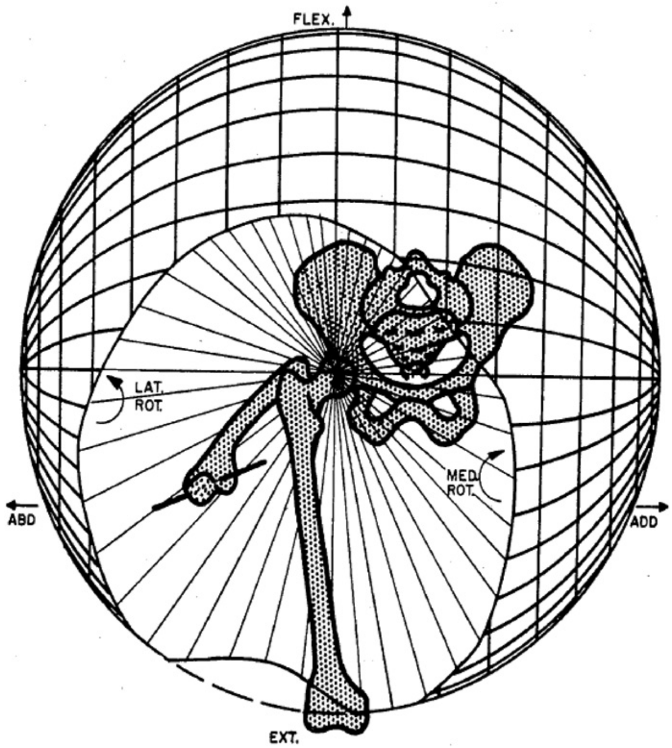

Fig. 1. Strasser's globographic presentation of hip joint movement (Strasser, 1917) redrawn and modified by Dempster (1956).

system and *globe* system. In contrast to Dempster and the early pioneers, most recent work has assumed that this method is primarily applicable to the shoulder.

All three methods can be implemented in a number of different ways resulting in different sets of three angles. This is referred to as sequence dependence for Euler/Cardan angles. Grood and Suntay (1983) claimed that the JCS "is sequence independent and eliminates much of the confusion in the nomenclature". This is a little disingenuous as the choice of different axes as fixed in the proximal and distal segments leads to different angles in exactly the same way, this will be referred to as "configuration dependence". The analogue for the globographic method is the choice of a primary axis and the orientation of the reference globe; this will be referred to as "orientation dependence".

The ISB standard based on the JCS makes an implicit assumption that essentially the same configuration is appropriate for all joints. This has been questioned by Baker (2003) who, in an earlier article (Baker, 2001), had suggested a different convention for the describing the pelvis in relation to a global axis system. He also questioned whether this was appropriate for the ankle. The basis for this is an assumption that whilst remaining biomechanically rigorous joint angle conventions should reflect conventional clinical definitions of terms as closely as possible. One particular problem with the JCS and Cardan/Euler angle approach is that they are generally described in abstract mathematical terms and little attempt is made to visualise what the angles represent. Without this understanding it is difficult to engage in dialogue with clinicians as to whether the definitions agree with conventional clinical usage or not. The globographic method, however, is inherently visual being analogous with widely understood conventions for establishing location and bearing on the surface of a globe.

A recent paper by Rab (2008) has shown that the underlying mathematics for ISB and globographic methods is identical for the shoulder joint. The aim of this paper is to explore the relationship between the two in more general terms and to include consideration of Cardan angles.

2. Mathematical background

The relative orientation of the distal CS (unit vectors $\hat{x}_d$, $\hat{y}_d$, $\hat{z}_d$) with respect to the proximal CS (unit vectors $\hat{x}_p$, $\hat{y}_p$, $\hat{z}_p$) is defined by the rotation matrix, R. This is the matrix that maps the coordinates, p_d, of a point within the distal CS to those, p_p, within the proximal CS and is formed of the dot products of all nine combinations of unit vectors[3]

$$p_p = Rp_d$$

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} \hat{x}_d\hat{x}_p & \hat{x}_d\hat{y}_p & \hat{x}_d\hat{z}_p \\ \hat{y}_d\hat{x}_p & \hat{y}_d\hat{y}_p & \hat{y}_d\hat{z}_p \\ \hat{z}_d\hat{x}_p & \hat{z}_d\hat{y}_p & \hat{z}_d\hat{z}_p \end{pmatrix} \quad (1)$$

The knee will be used as an example for all three systems as there is general agreement on the most appropriate application of all three approaches. We adopt the ISB convention for labelling axes (Wu and Cavanagh, 1995) and, using a right hand convention for defining angles for the right side, extension (Ex), adduction (Ad) and internal rotation (IR) are positive rotations.

2.1. Cardan/Euler angles

Fig. 2 depicts the generally accepted Cardan/Euler angles for the knee. Fig. 2a shows the result of rotating a CS originally aligned with that of the proximal segment about the lateral axis ($\hat{z}_p$) representing extension/flexion. Fig. 2b shows the result of rotating this about its anterior axis ($\hat{x}'$) representing ad/abduction and Fig. 1c shows the result of rotating this about its proximal axis ($\hat{y}''$). The first rotation is about $\hat{z}_p$ by definition. $\hat{y}''$ is invariant under the third rotation so, must already lie along $\hat{y}_d$ at the end of the second rotation. The second rotation is about an axis that is orthogonal to $\hat{z}_p$ (Fig. 2b) and $\hat{y}_d$ (Fig. 2d) and this mutual perpendicular is uniquely defined. Specifying the order in which the rotations are to be performed thus leads to a unique[4] set of three angles to describe relative orientation.

The advantage of Cardan/Euler angles is that the mathematics is straightforward. A rotation about one of the unit vectors x, y or z through a given angle α, β or γ is given by a matrix with a simple form:

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\alpha & s\alpha \\ 0 & -s\alpha & c\alpha \end{pmatrix}, \quad R_y(\beta) = \begin{pmatrix} c\beta & 0 & -s\beta \\ 0 & 1 & 0 \\ s\beta & 0 & c\beta \end{pmatrix},$$

$$R_z(\gamma) = \begin{pmatrix} c\gamma & s\gamma & 0 \\ -s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where c and s represent the cosine and sine functions, respectively:

$$R = R_y(IR)R_x(Ad)R_z(Ex)$$

$$= \begin{pmatrix} cIR\,cEx - sIR\,sAd\,sEx & cIR\,sEx + sIRs\,Ad\,cEx & -sIR\,cAd \\ -cAD\,sEx & cAd.cEx & sAd \\ sIR\,cEx + cIR\,sAD\,sEx & sIR\,sEx - cIR\,sAd\,cEx & cIR\,cAd \end{pmatrix}$$

[3] There is some inconsistency in the literature as to whether this matrix or its transpose should be considered as the rotation matrix. This paper follows the approach of Greenwood (2006), where the derivation from first principles is clearly outlined.

[4] Actually there are two solutions but we choose the one in which the middle rotation, adduction in this case, lies between -90° and $+90^\circ$.