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Short communication

A new dimensionless number highlighted from mechanical energy exchange during running

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ABSTRACT

This study aimed to highlight a new dimensionless number from mechanical energy transfer occurring at the centre of gravity (Cg) during running. We built two different-sized spring–mass models (SMM #1 and SMM #2). SMM #1 was built from the previously published data, and SMM #2 was built to be dynamically similar to SMM #1. The potential gravitational energy (E_P), kinetic energy (E_K), and potential elastic energy (E_E) were taken into account to test our hypothesis. For both SMM #1 and SMM #2, $N_{Mo-Dela} = (E_P + E_K)/E_E$ reached the same mean value and was constant (4.1 ± 0.7) between 30% and 70% of contact time. Values of $N_{Mo-Dela}$ obtained out of this time interval were due to the absence of E_E at initial and final times of the simulation. This phenomenon does not occur during *in vivo* running because a leg muscle's pre-activation enables potential elastic energy storage prior to ground contact. Our findings also revealed that two different-sized spring–mass models bouncing with equal $N_{Mo-Dela}$ values moved in a dynamically similar fashion. $N_{Mo-Dela}$, which can be expressed by the combination of Strouhal and Froude numbers, could be of great interest in order to study animal and human locomotion under Earth's gravity or to induce dynamic similarity between different-sized individuals during bouncing gaits.

1. Background

Dynamic similarity is a widely used concept in physics and engineering, which has also been applied to locomotion (Alexander and Jayes, 1983). This concept is an extension of the geometric similarity concept and it states that two systems are dynamically similar when a scale factor for lengths (C_L), another scale factor for masses (C_M), and a third scale factor (C_T) for times can be determined. This scaling principle appears to be very useful in making quantitative comparisons between individuals of a wide range in sizes (Alexander, 1984, 1989; DeJaeger et al., 2001; Minetti et al., 2000), and can lead to lower inter-individual variability (Bisiaux et al., 2003; Moretto et al., 2007). Dynamic similarity between two systems is possible only in particular conditions, which depend on the nature of the forces involved. For example, because gravitational forces are important during walking, dynamic similarity between two individuals requires them to have equal values of the Froude number $Nfr = v^2/gl$ (with v: forward speed; g: gravitational acceleration, l: leg length). Nfr was highlighted from the mechanical energy exchange occurring at the centre of gravity of an inverted pendulum used to model walking gait (Cavagna et al., 1977, 1976). Typically during a cycle of walking, the potential energy ($E_P = mgh$ with m: body mass; g: gravitational acceleration; h: height of the centre of gravity) and the kinetic energy ($E_K = 0.5mv^2$, with v: forward speed) have the same magnitude and are out of phase (Cavagna and Margaria, 1966; Farley and Ferris, 1998; Segers et al., 2007). The ratio of these two energies is thus constant, and reduces to Nfr, which represents the dimensionless expression of the speed. Equal values of Nfr were shown to ensure dynamic similarity between humans during walking (Bisiaux et al., 2003; Moretto et al., 2007), and between different animal species during running (Farley et al., 1993). As a matter of principle, because Nfr is based on the pendulum-like mechanics of walking, it does not take any elastic phenomenon into account, and thus seems too limited to study the dynamic similarity during running (Bullimore and Donelan, 2008).

In addition to potential gravitational energy (E_P) and kinetic energy (E_K), elastic energy ($E_E = 0.5k\Delta l^2$, with k: leg stiffness; Δl: variation in leg length) was shown to have an important role in mechanical energy conservation during running (Cavagna et al., 1964).

Taking the non-linear stress–strain relationship of the tendon into account may contribute to reducing deviations from dynamic similarity that occur when using Nfr during bouncing gaits

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(Bullimore and Burn, 2006). The spring–mass model enables us to take E_E into account, and is widely used in the literature to describe the mechanics of running. Considering that two runners behave in the same manner as such a vibrating system, Alexander (1989) suggested that dynamic similarity between them requires both to have the same values of the Strouhal number $Str =$ (frequency $\times$ length/speed). *Str* represents a dimensionless frequency, and in its expression, while the leg length appears to be the only length that can be used, in fact any frequency and any speed component of the centre of gravity (Cg) can be used. *Str*, including stride frequency and forward speed, induces inter-subject similarity exclusively for temporal parameters (Delattre et al., 2007). Alexander (1989) suggested that dynamic similarity between two runners requires concomitant equal values of *Nfr* and equal values of *Str*.

It arises that neither *Nfr* nor *Str* has been used for perfect dynamic similarity during running. *Nfr* seems to be relevant for dynamic similarity during walking, because it is based on mechanical energy transfer occurring in the pendulum-like mechanics of walking. To the best of our knowledge, there are no available studies dealing with dynamic similarity focusing on mechanical energy transfer of a spring–mass model.

This study aimed to consider mechanical energy transfer at the Cg of a simulated spring–mass model in order to highlight a new dimensionless number applicable for dynamic similarity during running. During the stance phase of running, the E_P and E_K variations are in phase, whereas E_K and E_E are out of phase (Biewener, 2006; Cavagna et al., 1964; Lee and Farley, 1998). Therefore, the $(E_P{+}E_K)/E_E$ ratio is expected to be constant during the stance phase of running. We suggest calling this ratio the Moretto–Delattre number ($N_{\mathrm{Mo-Dela}}$) in reference to its authors. As for *Nfr* during walking, $N_{\mathrm{Mo-Dela}}$ will have (i) to be constant during the running cycle and (ii) to reach the same value for two dynamically similar runners. The aim of this study is to verify these two hypotheses using two proportional spring–mass models set in similar dynamic conditions.

2. Methods

We created a computer simulation with Working Model 2D 7.0 (Design Simulation Technologies, Inc., USA). We built a first spring–mass model (SMM #1) using data from Ferris et al. (1999) (Table 1). We then built a second spring–mass model (SMM #2) dynamically similar to the first. The inter-model scale factors for length (L), mass (M), and time (T) dimensions were C_L, C_M, and C_T, respectively. They were determined as follows.

Table 1
Initial parameters of SMM #1 and the computation of the scale factors to determine the initial parameters of SMM #2

Initial parameters	Unit	Dimension	Scale factor between SMM #1 and SMM #2	SMM #1	SMM #2
l_{ini}	m	L	$C_L = 1.13$	0.87	0.983
m	kg	M	$C_M = 1.45$	53	76.85
Time	s	T	$C_T = 1.06$		
k	$\mathrm{kN\,m^{-1}}$	MT^{-2}	$C_M C_T^{-2} = 1.28$	6.9*	8.83
θ_{ini}	deg		1	26.6	26.6
Vx_{ini}	$\mathrm{m\,s^{-1}}$	LT^{-1}	$C_L C_T^{-1} = 1.06$	3	3.18
Vy_{ini}	$\mathrm{m\,s^{-1}}$	LT^{-1}	$C_L C_T^{-1} = 1.06$	−1	−1.06

l: spring length, m: mass, k: spring stiffness, θ: angle of the spring to the vertical, Vx: horizontal speed, Vy: vertical speed, C_L: scale factor for lengths, C_M: scale factor for masses, C_T: scale factor for times. Parameters of SMM #1 were those of Ferris et al. (1999) with *: hard surface with adjusted leg stiffness. In dynamically similar conditions, the scale factor for angles is equal to 1, meaning that angles between systems are identical in the same phase of the movement (Moretto et al., 2007).

The initial spring length of SMM #2 was set at a realistic value of 0.983 m, which implied that C_L equalled 1.13. Assuming that body densities of the two models were identical, the scale factor for masses was $C_M = C_L^3 = 1.45$, and thus the mass of SMM #2 was 76.85 kg (Table 1). The scale factor for times was determined as follows: E_P and E_K have the same dimension ML^2T^{-2}, and thus the same inter-subject scale factors. On the one hand, the ratio between E_{P1} and E_{P2} of two subjects S_1 and S_2 involves the simplification of the constant g, and is equivalent to the ratio of masses that multiplies the ratio of lengths, i.e. $C_M C_L$. Furthermore, E_K equals $0.5mv^2$, and has ML^2T^{-2} as a dimension, thus implying that $C_M C_L^2 C_T^{-2}$ is the scale factor for kinetic energies. Scale factors for E_P and E_K are the same, consequently

$$C_M C_L = C_M C_L^2 C_T^{-2} \Leftrightarrow C_L C_L^{-2} = C_T^{-2} \Leftrightarrow C_T^{-2} = C_L^{-1} \Leftrightarrow C_T = C_L^{0.5}$$

The scale factor for times (C_T) corresponds to the square root of the scale factor for lengths (C_L) and thus equals 1.06 (Table 1).

L, M, and T are fundamental dimensions; thus all mechanical parameters can be defined from them. For example, the spring stiffness k is expressed in $\mathrm{N\,m^{-1}}$ (or $\mathrm{kg\,m\,s^{-2}\,m^{-1}}$) and its dimension is MT^{-2}. Accordingly, between two subjects moving in a dynamically similar fashion, the scale factor for the stiffness (C_k) equals the product of scale factors involved: $C_k = C_M C_T^{-2}$. In the same way, the scale factors for the other parameters of the simulations (horizontal and vertical speeds) are computed from C_L, C_M, and C_T. Table 1 summarizes initial parameters for simulations SMM #1 and SMM #2, and Fig. 1 shows the initial configuration of the two simulations.

Data from each simulation enabled us to compute the gravitational potential energy $E_P = mgh$, the kinetic energy $E_K = 0.5mv^2$, and the elastic energy $E_E = 0.5k\,\Delta l^2$ (with m: mass; g: gravitational acceleration; h: vertical position of Cg; v: speed; k: spring stiffness; Δl: spring compression) and their temporal evolutions.

3. Results

For each of the two models, θ and l had the same value at the initial (touchdown) and at the final (toe-off) times of the simulation, and θ equalled zero degrees when the spring was maximally compressed (Table 2).

Table 3 shows that scale factors calculated from computed parameters matched the scale factors predicted from C_L, C_M, and

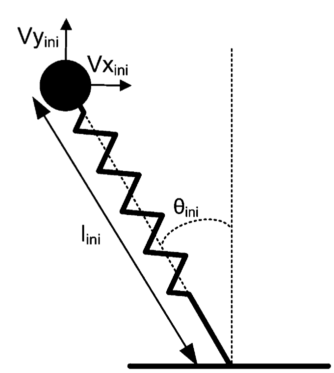

Fig. 1. Initial configuration of the two simulated spring–mass models SMM #1 and SMM #2. l_{ini}: initial spring length; θ_{ini}: initial angle of the spring to the vertical; Vx_{ini}: initial horizontal speed; Vy_{ini}: initial vertical speed.

Table 2
Values of θ and l at key times of the two simulations SMM #1 and SMM #2

Parameter (unit)	SMM #1	SMM #2
θ_{ini} (deg)	26.6	26.6
θ_{fin} (deg)	26.6	26.6
$\theta_{\Delta l\,max}$ (deg)	0	0
l_{ini} (m)	0.87	0.985
l_{fin} (m)	0.87	0.985

Ini: values at the initial time of the simulation; fin: values at the final time of the simulation; $\theta_{\Delta l\,max}$: angle of spring to the vertical when the spring is maximally compressed.
Results presented in this table show that the two models bounced symmetrically.