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Short communication

A visco-hyperelastic model for skeletal muscle tissue under high strain rates

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ABSTRACT

In this paper, a visco-hyperelastic skeletal muscle model is developed. The constitutive relation is based on the definition of a Helmholtz free energy function. It is assumed that the Helmholtz energy can be decomposed into volumetric and isochoric parts; furthermore, the isochoric energy can be decoupled into hyperelastic and viscous parts. The model developed involves 14 material parameters and its performance is evaluated by comparing the finite element simulation results with the published experimental studies on the New Zealand white rabbit tibialis anterior muscle. Results show that this model is able to describe the visco-hyperelastic behaviour of both passive and active skeletal muscle tissues under high strain rates (10/s and 25/s).

1. Introduction

Skeletal muscle plays an important role in the human body system. It generates voluntary forces leading to motion and provides protection to the upright skeleton. Like other biological tissues, it exhibits a highly complex mechanical behaviour which includes active, quasi-incompressible, fibre-enforced, viscoelastic and hyperelastic behaviour (Fung, 1981). Although some mathematical skeletal muscle models (Blemker et al., 2005; Martins et al., 2006; Tang et al., 2009) have been developed in recent years, the effect of viscoelasticity has been ignored in these models. However, it has been shown that the stress–strain relationship of skeletal muscle is very sensitive to the loading rates (Myers et al., 1998; Van Loocke et al., 2008). Furthermore, in the injury simulations of human body involved in car crash and sports impact, the viscoelastic properties of the human muscles certainly play an important role, since they are loading rate-dependent. Thus, it is very demanding to develop accurate visco-hyperelastic skeletal muscle models.

Only a few viscoelastic skeletal muscle models have been developed in recent years. Tsui et al. (2004) used a viscoelastic motor element, which was represented by a linear time function of the strain rate and activation amplitude. Hedenstierna et al. (2008) simulated the muscle tissue behaviour by combining passive non-linear viscoelastic continuum elements with active truss elements. Van Loocke et al. (2008) proposed a quasi-linear viscoelastic (QLV), strain-dependent Young's moduli (SYM) model to represent the skeletal muscle viscoelastic behaviour in compression. However, it was found that this QLV–SYM model presented poorer predictive capabilities due to nonlinearities in the tissue response. A new nonlinear viscoelastic (NLV), strain-dependent Young's moduli (SYM) model (Van Loocke et al., 2009) was developed to capture the nonlinear behaviour observed during low frequency, high amplitude cyclic tests. In this paper, an alternative method is used to develop the visco-hyperelastic skeletal muscle constitutive relation. This method uses the definition of a Helmholtz free energy function (Ogden, 1984) and it has been used to develop visco-hyperelastic constitutive relations for human knee ligaments (Pioletti et al., 1998), biological connective tissues (Limbert and Middleton, 2004), posterior cruciate ligament (Limbert and Middleton, 2006) and periodontal ligament (Zhurov et al., 2007). However, its application to skeletal muscle tissue has not been investigated to date. In this paper, the visco-hyperelastic constitutive relation for skeletal muscle is formulated and the performance of this model is evaluated.

2. Constitutive relation for visco-hyperelastic skeletal muscle

The skeletal muscle is modelled as an active, quasi-incompressible, transversely isotropic and visco-hyperelastic composite, comprising a ground substance and the muscle fibres (Fig. 1). The ground substance is assumed to be a compliable isotropic solid. The fibres are assumed to have a single preferred direction $\mathbf{n}_0$ and modelled using Hill's 3-Element model (Fig. 1). Both the matrix and the fibres are assumed to have a viscous response.

It is assumed that the Helmholtz energy function can be decoupled into an elastic energy function and a viscous energy function. To describe a quasi-incompressible solid, an additional decomposition of the strain energy function into a volumetric and an isochoric part is performed. The total strain energy in the

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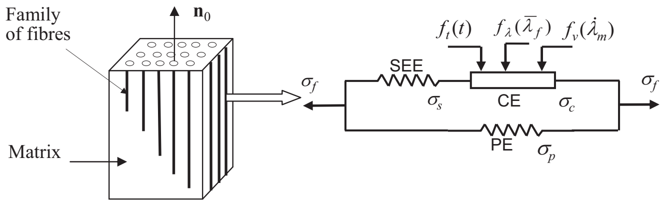

Fig. 1. Simplified representation of a skeletal muscle.

skeletal muscle can be written as

$$\psi=\psi_{\mathrm{iso}}^{e}\left[\left\{\bar{I}_{\alpha}(\mathbf{X},\bar{\mathbf{C}},\mathbf{N})\right\}_{\alpha=1\ldots5}\right]+\psi_{\mathrm{iso}}^{v}\left[\left\{\bar{J}_{\alpha}(\mathbf{X},\bar{\mathbf{C}},\dot{\bar{\mathbf{C}}},\mathbf{N})_{\alpha=1\ldots12}\right\}\right]+\psi_{\mathrm{vol}}(J) \tag{1}$$

where $\mathbf{X}$ is the position of the material point in the reference configuration, $\bar{\mathbf{C}}$ is the modified right Cauchy–Green deformation tensor, $\dot{\bar{\mathbf{C}}}$ is the modified rate of the right Cauchy–Green deformation tensor, $\mathbf{N}$ is a symmetric second order tensor $\mathbf{N}=\mathbf{n}_0\otimes\mathbf{n}_0$, $\{\bar{I}_\alpha\}_{\alpha=1\ldots5}$ and $\{\bar{J}_\alpha\}_{\alpha=1\ldots12}$ are the modified invariants and J is the Jacobean of the deformation gradient.

2.1. Volumetric response function ψ_{vol}

The quadratic volumetric response function used by Martins et al. (1998) is adopted here:

$$\psi_{\mathrm{vol}}(J)=\frac{1}{D}(J-1)^2 \tag{2}$$

where D is the compressibility constant.

2.2. Isochoric hyperelastic strain energy function ψ_{iso}^{e}

The isochoric hyperelastic contributions come from both the matrix and muscle fibres. It is assumed that the hyperelastic interaction between the matrix and muscle fibres can be ignored. Therefore, the isochoric hyperelastic strain energy can be written as

$$\psi_{\mathrm{iso}}^{e}=\psi_{m}^{e}(\bar{I}_1)+\psi_{f}^{e}(\bar{\lambda}_f,\lambda_s) \tag{3}$$

The hyperelastic matrix contribution is characterised with the neo–Hookean strain energy density (Ogden, 1984)

$$\psi_{m}^{e}(\bar{I}_1)=\frac{b}{2}(\bar{I}_1-3) \tag{4}$$

where $\bar{I}_1=\mathbf{I}{:}\bar{\mathbf{C}}$, $\mathbf{I}$ is the second order unit tensor and b is a material parameter.

The hyperelastic response from the fibres is characterised using Hill's 3-element model (Fig. 1). The total stress in the fibre σ_f is the sum of the stresses in the series elastic element (SEE) σ_s and the parallel element (PE) σ_p. The model presented in this paper is mainly used to characterise the skeletal muscle behaviour under tension. So, when the muscle is under the passive and compressive state, the assumption that the fibres exhibit zero force leading to an isotrpic model is made, although it has been demonstrated in Van Loocke et al. (2006) work that skeletal muscle exhibits non-isotropic in compression. The hyperelastic energy function for the fibres can be expressed as

$$\psi_{f}^{e}(\bar{\lambda}_f,\lambda_s)=\int_{1}^{\bar{\lambda}_f}[\sigma_s(\lambda,\lambda_s)+\sigma_p(\lambda)]\,d\lambda \tag{5}$$

where $\bar{\lambda}_f$ is the fibre stretch ratio and λ_s is the stretch ratio in the SEE.

A recurrence relation is used to express the stress produced in the SEE (Fung, 1981):

$$^{t+\Delta t}\sigma_s=e^{\alpha\Delta\lambda_s}(^{t}\sigma_s+\beta)-\beta \tag{6}$$

where

$$^{t}\sigma_s=\beta[e^{\alpha(^{t}\lambda_s}-1)-1] \tag{7}$$

and α and β are material constants.

The stress produced in the contractile element (CE) is given by

$$^{t+\Delta t}\sigma_m=\sigma_0 f_t(t+\Delta t)f_\lambda(\bar{\lambda}_f)f_v(\dot{\lambda}_m) \tag{8}$$

where

$$f_t(t)=\begin{cases}n_1, & \text{if } t<t_0\\ n_1+(n_2-n_1)h_t(t,t_0), & \text{if } t_0<t<t_1\\ n_1+(n_2-n_1)h_t(t_1,t_0)-[(n_2-n_1)h_t(t_1,t_0)]h_t(t,t_1), & \text{if } t>t_1\end{cases} \tag{9}$$

$$h_t(t_i,t_b)=\{1-\exp[-S\cdot(t_i-t_b)]\}, \tag{10}$$

is the muscle activation function;

$$f_\lambda(^{t}\bar{\lambda}_f)=\begin{cases}0, & \text{if } ^{t}\bar{\lambda}_f/\lambda_{opt}<0.4\\ 9(^{t}\bar{\lambda}_f/\lambda_{opt}-0.4)^2, & \text{if } 0.4\le {}^{t}\bar{\lambda}_f/\lambda_{opt}<0.6\\ 1-4(1-{}^{t}\bar{\lambda}_f/\lambda_{opt})^2, & \text{if } 0.6\le {}^{t}\bar{\lambda}_f/\lambda_{opt}<1.4\\ 9(^{t}\bar{\lambda}_f/\lambda_{opt}-1.6)^2, & \text{if } 1.4\le {}^{t}\bar{\lambda}_f/\lambda_{opt}<1.6\\ 0, & \text{if } ^{t}\bar{\lambda}_f/\lambda_{opt}\ge 1.6\end{cases} \tag{11}$$

is the muscle stress–stretch function and

$$f_v(\dot{\lambda}_m)=\begin{cases}\dfrac{1-\dot{\lambda}_m/\dot{\lambda}_m^{\min}}{1+k_c\dot{\lambda}_m/\dot{\lambda}_m^{\min}}, & \text{if } \dot{\lambda}_m\le 0\\ d-(d-1)\dfrac{1+\dot{\lambda}_m/\dot{\lambda}_m^{\min}}{1-k_ck_e\dot{\lambda}_m/\dot{\lambda}_m^{\min}}, & \text{if } \dot{\lambda}_m>0\end{cases} \tag{12}$$

is the muscle stress–velocity function.

In these definitions, σ_0 is the maximum isometric stress, n_1 is the activation level before and after the activation, n_2 is the activation level during the activation, t_0 is the activation time, t_1 is the deactivation time, S is the exponential factor, λ_{opt} is the optimal fibre stretch at which the sarcomere reaches its optimal length, k_c and k_e are the shape parameters of the hyperbolic curves, d is the offset of the eccentric function, $\dot{\lambda}_m$ is the stretch rate in CE and $\dot{\lambda}_m^{\min}$ is the minimum stretch rate.

Eq. (6) contains one unknown, namely $\Delta\lambda_s$, and this can be solved by setting up a non-linear equation utilizing the stress relationship between the CE and the SEE (Tang et al., 2009).

The stress in the PE is expressed as

$$^{t+\Delta t}\sigma_p=\sigma_0 f_{PE}(^{t+\Delta t}\bar{\lambda}_f) \tag{13}$$

where

$$f_{PE}(^{t+\Delta t}\bar{\lambda}_f)=\begin{cases}A(^{t+\Delta t}\bar{\lambda}_f-1), & \text{if } ^{t+\Delta t}\bar{\lambda}_f>1\\ 0, & \text{otherwise}\end{cases} \tag{14}$$

and A is a material parameter.

2.3. Isochoric viscous strain energy function ψ^v

The viscous response of the material is assumed to be provided by the matrix and muscle fibres. The formation, which is used by Limbert and Middleton (2004) and Zhurov et al. (2007) for viscous contribution, is employed here

$$\psi_{\mathrm{iso}}^{v}(\bar{I}_1\bar{J}_2,\bar{I}_4\bar{J}_5)=\psi_{m}^{v}(\bar{I}_1\bar{J}_2)+\psi_{f}^{v}(\bar{I}_4\bar{J}_5) \tag{15}$$

with

$$\psi_{m}^{v}(\bar{I}_1\bar{J}_2)=\eta_1(\bar{I}_1-3)\bar{J}_2 \tag{16}$$