Infectious Disease Modelling 3 (2018) 1–12

Contents lists available at ScienceDirect

Infectious Disease Modelling

journal homepage: www.keaipublishing.com/idm

Relatedness of the incidence decay with exponential adjustment (IDEA) model, "Farr's law" and SIR compartmental difference equation models

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ARTICLE INFO

Article history:
Received 27 February 2018
Accepted 2 March 2018
Available online 9 March 2018

ABSTRACT

Mathematical models are often regarded as recent innovations in the description and analysis of infectious disease outbreaks and epidemics, but simple mathematical expressions have been in use for projection of epidemic trajectories for more than a century. We recently introduced a single equation model (the incidence decay with exponential adjustment, or IDEA model) that can be used for short-term epidemiological forecasting. In the mid-19th century, Dr. William Farr made the observation that epidemic events rise and fall in a roughly symmetrical pattern that can be approximated by a bell-shaped curve. He noticed that this time-evolution behavior could be captured by a single mathematical formula ("Farr's law") that could be used for epidemic forecasting. We show here that the IDEA model follows Farr's law, and show that for intuitive assumptions, Farr's Law can be derived from the IDEA model. Moreover, we show that both mathematical approaches, Farr's Law and the IDEA model, resemble solutions of a susceptible-infectious-removed (SIR) compartmental differential-equation model in an asymptotic limit, where the changes of disease transmission respond to control measures, and not only to the depletion of susceptible individuals. This suggests that the concept of the reproduction number ($\mathscr{R}_0$) was implicitly captured in Farr's (pre-microbial era) work, and also suggests that control of epidemics, whether via behavior change or intervention, is as integral to the natural history of epidemics as is the dynamics of disease transmission.

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Peer review under responsibility of KeAi Communications Co., Ltd.

https://doi.org/10.1016/j.idm.2018.03.001

1. Introduction

The control of communicable diseases is an endeavor that has witnessed remarkable successes over the past century; diseases that previously caused large scale mortality have been eradicated (Hinman, 1999; Roeder, Mariner, & Kock, 2013), locally eliminated (Papania et al., 2014), or have been markedly reduced in incidence globally as a result of vaccination, antimicrobial therapy, water and sewage treatment, and advances in food safety (Armstrong, Conn, & Pinner, 1999; Liu et al., 2015; Murray et al., 2014). Nonetheless, the threat of communicable diseases persists; emerging infectious diseases continue to be identified, often in association with changes in human and animal mobility, agricultural practices, environmental degradation, and misuse of antimicrobial therapy (Jones et al., 2008; Keesing et al., 2010; Kuehn, 2010). Recent outbreaks or epidemics associated with MERS coronavirus (Azhar et al., 2014), influenza A (H7N9) (Cowling et al., 2013), and the West African emergence of the Zaire strain of Ebola virus (Baize et al., 2014), have challenged epidemiologists as the natural history, modes of transmission, and/or means of control of these diseases have not been well understood during initial periods of emergence.

When novel infectious diseases emerge or familiar diseases resurge, mathematical models can serve as useful tools for the synthesis of available data, management of uncertainty, and projection of likely epidemic trajectories (Fisman, 2009). While it may be challenging to parameterize detailed mechanistic mathematical models when there is little information on mechanisms of transmission, baseline immunity in a community, or the nature of the infecting pathogen, a number of descriptive approaches exist which may permit fitting, and forecasting, of an epidemic curve. One single equation approach that has been applied to emerging infections is the Richards model, which treats cumulative infections as a logistic growth process (Hsieh & Chen, 2009; Wang, Wu, & Yang, 2012). However, the concept of modeling an epidemic curve as a simple function, without reference to mechanisms of transmission, is in fact much older, and may originate in the work of the English polymath Dr. William Farr (1807–1883), who rose from humble beginnings to become a physician, mathematician, hygienist and protege of Lancet founder Dr. Thomas Wakley (Brownlee, 1915a; Fine, 1979; Greenwood, 1933). Dr. Farr spent almost 40 years at the General Register Office of the United Kingdom, and the esteem in which he was held is apparent in the "letters" he published annually as appendices to the reports of the Registrar General, in which he supplemented the dry statistical reports with thoughtful and creative musings on topics as wide-ranging as the relationships between occupation and disease, suicide and mortality in the mentally ill, population density and mortality, and as above the "laws" governing epidemics (Farr, 1840).

William Farr's analysis is a classic in the epidemiology literature. Farr examined the course of mortality attributable to smallpox between mid 1837 (when death registration was introduced into England and Wales) and 1839, and noted that the numbers peaked in the spring quarter of 1838 and then declined until summer 1839 (Fig. 1) (Farr, 1840). He noted that the pattern of decline was very close to what would be predicted if the ratios of cases in successive quarters declined at a constant rate. He provided numbers demonstrating this in his annual report to the Registrar General in 1840 (Fig. 1), but did not develop the idea at length. Looking back on this, we may note that this approach is analogous to assuming that the number of transmissions per case (or the "reproduction number" in modern terminology), were to decline at a constant rate during the course of an epidemic. The key difference is that Farr worked before the germ theory, and analysed data in terms of successive calendar time periods rather than successive generations of cases. There is a further irony to the story, in that he never returned to this idea until 1866, at which time there was a major epidemic of rinderpest, which some feared would destroy the British cattle population (Brownlee, 1915a). Farr applied a similar analysis, but this time based upon assuming that the third ratio of cases per month was a constant (in effect assuming that the reproduction number declined at a constantly accelerating pace). He used this approach to predict that the epidemic would decline rapidly over the subsequent six months, and published this, including predicted monthly incidence numbers, in the Daily News of London in February 1866. His predictions were close to what subsequently happened (Brownlee, 1915a).

It fell to other contemporaries (Evans, 1876) and later epidemiologists (most notably Dr. John Brownlee) to formalize "Farr's law" (Brownlee, 1915c; Fine, 1979; Serfling, 1952). (It should be noted that the term "Farr's law" is ambiguous. Farr himself referred to a "law" in his letter on rinderpest (Brownlee, 1915a), but the term has also been used by others to describe Farr's observations on the relation between population density and death (Brownlee, 1915b), and to his description of the relationship between cholera mortality and altitude (Lilienfeld, 2007). In his elaboration of the "law", Brownlee referred to it as "Farr's theory of epidemics" (Brownlee, 1915a)).

We recently proposed a descriptive approach to the initial estimation of the basic reproduction number ($\mathscr{R}_0$) of an emerging or re-emerging pathogen, which also provides information on the rate at which the process is being controlled, as well as reasonable short-term projections of incidence. This two-parameter model, which we have referred to as the "Incidence Decay with Exponential Adjustment" (IDEA) model, offers advantages of simplicity, explicit linkage to theory of epidemic growth, and also acknowledges the fact that epidemics and outbreaks do not peak and end simply due to depletion of susceptibles, but because of a complex constellation of public health actions and behavioral changes that may modify the course of an epidemic and reduce the effective reproduction number $\mathscr{R}_e(t)$ during an outbreak (Fisman et al., 2013). In our previously published description of this model, we validated model projections by showing that they were identical to those derived from a discrete-time susceptible-infectious-removed (SIR) compartmental model, provided the SIR model had a low basic reproduction number ($\mathscr{R}_0$) and exponential improvement in control over the course of the epidemic (Fisman et al., 2013).

One of us (PF) had previously written about Farr's law and its importance in the development of epidemic theory (Fine, 1979), and noted the conceptual similarity between IDEA and Farr's law. Upon exploration of these two approaches we