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Duration of a minor epidemic

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ABSTRACT

Disease outbreaks in stochastic SIR epidemic models are characterized as either minor or major. When $\mathcal{R}_0 < 1$, all epidemics are minor, whereas if $\mathcal{R}_0 > 1$, they can be minor or major. In 1955, Whittle derived formulas for the probability of a minor or a major epidemic. A minor epidemic is distinguished from a major one in that a minor epidemic is generally of shorter duration and has substantially fewer cases than a major epidemic. In this investigation, analytical formulas are derived that approximate the probability density, the mean, and the higher-order moments for the duration of a minor epidemic. These analytical results are applicable to minor epidemics in stochastic SIR, SIS, and SIRS models with a single infected class. The probability density for minor epidemics in more complex epidemic models can be computed numerically applying multitype branching processes and the backward Kolmogorov differential equations. When $\mathcal{R}_0$ is close to one, minor epidemics are more common than major epidemics and their duration is significantly longer than when $\mathcal{R}_0 \ll 1$ or $\mathcal{R}_0 \gg 1$.

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1. Introduction

Public health intervention and control strategies are designed to shorten the course of an epidemic. This is often achieved through reduction of the basic reproduction number $\mathcal{R}_0$ to a value below the critical threshold of one. However, for $\mathcal{R}_0$ close to one, the duration of an epidemic is increased. This property is demonstrated in the following study of the duration of minor epidemics in stochastic epidemic models.

It is well-known for deterministic SIR epidemic models that if $\mathcal{R}_0 < 1$, the number of infected cases decreases over time, whereas if $\mathcal{R}_0 > 1$, the number of cases increases. However, the outcome differs in stochastic SIR epidemic models. In 1955 Whittle characterized two different types of epidemics in stochastic SIR models as minor or major (Whittle, 1955). Only minor epidemics occur when $\mathcal{R}_0 < 1$ and either minor or major occur when $\mathcal{R}_0 > 1$. Minor epidemics are distinguished from major ones as generally being of shorter duration and with substantially fewer cases. Whittle derived the well-known formulas $(1/\mathcal{R}_0)^i$ for the probability of a minor epidemic and $1-(1/\mathcal{R}_0)^i$ for the probability of a major epidemic, given there are initially i infected individuals and $\mathcal{R}_0 > 1$. Analytical expressions for the duration of these minor outbreaks are not well known. In this investigation, analytical formulas are derived that approximate the probability density, the mean, and the

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higher-order moments for a minor epidemic of continuous-time Markov chain (CTMC) SIR, SIS and SIRS epidemic models. In particular, it is shown that the mean duration T of a minor epidemic given $I(0)=1$ is

$$\mathbb{E}(T|T<\infty)=\begin{cases}\dfrac{1}{\gamma}\ln\left(\dfrac{\mathcal{R}_0}{\mathcal{R}_0-1}\right), & \mathcal{R}_0>1,\\ \dfrac{1}{\beta}\ln\left(\dfrac{1}{1-\mathcal{R}_0}\right), & \mathcal{R}_0<1,\end{cases}$$

where β is the transmission rate, γ is the recovery rate, and $\mathcal{R}_0=\beta/\gamma$.

The probability of a minor or a major epidemic has been studied in more complex disease settings than the CTMC SIR epidemic model by approximating the dynamics with a simple birth-death process, the backward Kolmogorov differential equations, and a multitype branching process approximation (e.g. Allen, 2015; Allen, 2017; Allen & Lahodny, 2012; Allen & van den Driessche, 2013; Athreya & Ney, 1972; Griffiths, 1972; Griffiths, 1973). The probability estimates are generally better for large population sizes and large values of $\mathcal{R}_0$ (e.g. Allen, 2015; Allen, 2017; Allen & Lahodny, 2012; Allen & van den Driessche, 2013). Most theoretical research in stochastic models has focused on the duration of major SIR epidemics and quasistationarity in SIS models when conditioned on nonextinction (e.g. Artalejo, 2012; Daley & Gani, 1999; Hernandez-Ceron, Chavez-Casillas, & Feng, 2015; Hernández-Suárez & Castillo-Chavez, 1999; Kryscio & Lefévre, 1989; Norden, 1982; Nåsell, 1996; Nåsell, 1999; Nåsell, 2001; van Doorn, 1991). In this investigation, the emphasis is on the duration of minor epidemics, conditioned on extinction.

In the following section, the simple birth-death process and branching process theory are used to derive the probability density and analytical formulas for the moments for time to extinction. In Section 3, the results from the simple birth-death process are applied to minor epidemics in CTMC SIR and SIS models. Numerical results show good agreement between the simulations and analytical estimates, given the population size is large. The increase in duration when $\mathcal{R}_0$ is close to one is demonstrated in these models and in Section 4 as well for the SEIR model with a latent stage.

2. Simple birth-death process

The simple birth-death process is a well-known approximation for many population processes (see e.g. Allen, 2010; Athreya & Ney, 1972; Bailey, 1975; Daley & Gani, 1999; Novozhilov, Karev, & Koonin, 2006; Sehl, Zhou, Sinsheimer, & Lange, 2011; Whittle, 1955). The mean of the simple birth-death process is an exponential growth model,

$$\frac{dm}{dt}=(B-D)m, \tag{1}$$

where B and D are the per capita birth and death rates, respectively. We summarize briefly the simple birth-death process and the expressions that lead to the probability of extinction and the time to extinction.

Let $X(t)\in\{0,1,2,\ldots\}$ denote the discrete random variable for the population size in a time-homogeneous process with transition probabilities defined as

$$p_{i,j}(t-s)=\mathbb{P}(X(t)=j|X(s)=i).$$

The infinitesimal transition probabilities for a small period of time Δt can be expressed as

$$p_{i,j}(\Delta t)=\mathbb{P}(X(t+\Delta t)=j|X(t)=i)=\begin{cases}Bi\Delta t+o(\Delta t), & j=i+1,\\ Di\Delta t+o(\Delta t), & j=i-1,\\ 1-(B+D)i\Delta t+o(\Delta t), & j=i,\\ o(\Delta t), & j\neq i-1,i,i+1.\end{cases} \tag{2}$$

The expressions in (2) lead to the backward Kolmogorov differential equations

$$\frac{dp_{i,j}}{dt}=Bip_{i+1,j}+Dip_{i-1,j}-(B+D)ip_{i,j}. \tag{3}$$

An important assumption in the simple birth-death process is that births and deaths occur independently of each other. Therefore, the probability of extinction at time t beginning from i individuals, $p_{i,0}(t)$, can be written as $(p_{1,0}(t))^i$. Derivation of the expression for $p_{i,0}(t)$ follows from generating functions and branching process theory (Athreya & Ney, 1972; Bailey, 1975; Daley & Gani, 1999; Harris, 1963):

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