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Substructuring and poroelastic modelling of the intervertebral disc

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ABSTRACT

We proposed a substructure technique to predict the time-dependant response of biological tissue within the framework of a finite element resolution. Theoretical considerations in poroelasticity preceded the calculation of the sub-structured poroelastic matrix. The transient response was obtained using an exponential fitting method. We computed the creep response of an MRI 3D reconstructed L_5–S_1 intervertebral disc of a scoliotic spine. The FE model was reduced from 10,000 degrees of freedom for the full 3D disc to only 40 degrees of freedom for the sub-structured model defined by 10 nodes attached to junction nodes located on both lower and upper surfaces of the disc. Comparisons of displacement fields were made between the full poroelastic FE model and the sub-structured model in three different loading conditions: compression, offset compression and torsion. Discrepancies in displacement were lower than 10% for the first time steps when time-dependant events were significant. The substructuring technique provided an exact solution in quasi-static behavior after pressure relaxation. Couplings between vertical and transversal displacements predicted by the reference FE model were well stored by the sub-structured model despite the drastic reduction of degrees of freedom. Finally, we demonstrated that substructuring was very efficient to reduce the size of numerical models while respecting the time-dependant behavior of the structure. This result highlighted the potential interest of substructure techniques in large-scale models of musculoskeletal structures.

1. Introduction

Numerous computational models have been developed to investigate the biomechanical behavior of the spine (Stokes and Laible, 1990; Lavaste et al., 1992). Complex models involved non-linearities such as vertebrae fractures, contacts between articular facets, and non-linear behaviors of ligaments and muscles (Smit, 1996; Dolan and Adams, 2001; Imai et al., 2006; Noailly et al., 2007). Advanced models of intervertebral discs have also been implemented (Simon et al., 1985; Argoubi and Shirazi-Adl, 1996; Frijns et al., 1997; Martinez et al., 1997; Lee et al., 2000). The relevance of predictive models increased along with their complexity. However, it appears that management of non-linear behavior is computationally time consuming and it is still difficult to cumulate the description of local effects in predictive large-scale models. This difficulty has been particularly limitative to intervertebral disc modelling whereas disc local behavior and global behavior of the spine interact in the pathogenesis of spine (Périé et al., 2003; Brisby, 2006; Natarajan et al., 2006).

The substructuring technique associated with the finite element method proved to be of great interest in computational mechanics in linear statics and dynamics. The robustness of the method has been shown and its limitations have been discussed (Lalanne et al., 1983; Zienkiewicz et al., 2005). This method was used to predict the mechanical behavior of a complex structure by dividing the structure into a series of smaller structures, called substructures, and studying the intrinsic behavior of these components. Substructuring allowed a considerable reduction in the number of degrees of freedom, which would otherwise be required to model the entire initial structure. In linear behavior, the underlying assumption of this technique was that the superposition principle of elasticity was applicable. This technique has been adapted to cases where non-linear behavior was confined to certain parts of the structures, such stress concentrations, crack tips, non-linear material and mounts (Hellen, 1984; Gjika et al., 1996).

In our study, we hypothesized that the substructuring technique could be relevant in case of non-linear elastic structure with poroelastic behavior. The substructuring technique was applied to an intervertebral disc within sight, the potential use in larger computational model of vertebral segments. The creep response of a L_5–S_1 intervertebral disc reconstructed from MRI was investigated.

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Nomenclature

u^s	displacement of structural phase
u^f	displacement of fluid phase
u^{fs}	displacement of fluid vs structural phase
$\dot{u}, \ddot{u}$	velocity, acceleration
ϕ	tissue porosity
p	interstitial fluid pressure
σ_{ij}	stress tensor
ρ, ρ^s, ρ^f	densities
q^f	flow rate of the fluid phase
μ^f	fluid dynamic viscosity
k^s	compressibility of structural phase
k^f	compressibility of fluid phase
k^f	compressibility modulus of fluid
α	Biot coefficient
κ_{ij}	specific permeability tensor
ε, ξ	volume strain, permeability decay factor
K	structural stiffness matrix
M	mass matrix
Q	fluid-structure matrix
H	permeability matrix
C_t	tangent stiffness matrix,
B	strain–displacement matrix
N^u, N^p	shape functions
P	poroelastic matrix
f^u, f^p	structural and flux nodal forces
δ, δ^e, δ^i	dof, external dof, internal dof
$\overline{P}$	sub-structured poroelastic matrix
f	equivalent nodal force
f^u, f^p	nodal force vector, nodal fluid flux
u^e, p^e	external nodal displacement and pressure
u^i, p^i	internal nodal displacement and pressure
$\overline{u}^e$	displacements of sub-structured model
f^e, f^i	external and internal nodal forces
f^{ue}, f^{ui}	external and internal nodal forces
f^{pe}, f^{pi}	external and internal nodal flux
η	decay factor
$e_{\%}$	error criterion in displacement

2. Material and methods

2.1. Poroelastic governing equations

The continuum was modeled using two phases: the structural phase Ω^s and the fluid phase Ω^f. The conservation of momentum of fluid phase neglecting fluid velocity gradient led to the generalized Darcy Law (1) (Biot, 1941; Coussy, 1995; Meroi et al., 1999). The conservation of momentum for structural phase was expressed by Eq. (2), and the continuity equation of fluid phase was given by Eq. (3).

$$q_i^f = \phi \dot{u}_i^{fs} = \frac{\kappa_{ij}}{\mu_f}\left[-p_{,j} + \rho^f(g_j - \ddot{u}_j^s - \ddot{u}_j^{fs})\right] \tag{1}$$

$$\sigma_{ji,j} + \rho g_i = \rho \ddot{u}_i^s + \phi\rho^f(\ddot{u}_i^{fs} + \dot{u}_i^{fs}\dot{u}_{i,i}^{fs}) \tag{2}$$

$$\dot{u}_{i,i}^s + \left[\frac{(\alpha-\phi)}{k^s} + \frac{\phi}{k^f}\right]\dot{p} + q_{i,i}^f + q_i^f\frac{\rho_{,i}^f}{\rho^f} = 0 \tag{3}$$

Boundary conditions (4) were imposed displacements d and imposed force σ on (Γ^d,Γ^σ of the structural phase Ω^s. Imposed flux and imposed pressure on (Γ^q, Γ^p) concerned the fluid phase Ω^f.

$$u^s(x,t) = d \text{ for } x \in \Gamma^d \;\; \sigma(x,t)n = \sigma \text{ for } x \in \Gamma^\sigma \tag{4}$$

$$p(x,t) = p \text{ for } x \in \Gamma^p \;\; q_i^i n = q \text{ for } x \in \Gamma^q$$

Within the finite element resolution, the previous continuous problem was formulated using the matrix system (5) where nodal degrees of freedom (dof) were the displacement field u and the pressure field p (Pédrono, 2001). Related elementary matrices and vectors are briefly listed in Appendix.

$$\frac{1}{2}Ku^s - Qp - M\ddot{u}^s = f^u \text{ and } Hp + Q^t\dot{u}^s = f^p \tag{5}$$

Additional assumptions were made in the theoretical model. The structural phase Ω^s was isotropic, incompressible and involved small strains (k^s=+∞, α=1). The fluid phase Ω^f was incompressible ($k^f = +\infty, \rho_i^f = 0$). Filtration terms ($\ddot{u}_i^{fs}, \dot{u}_i^{fs}\dot{u}_{i,i}^{fs}$) and accelerations in the Darcy law were neglected (Coussy, 1995).

2.2. Adaptation of the substructuring technique

The resolution of system (5) was achieved using a linearization scheme in time and space. This was obtained using a time Newmark finite difference scheme of order 2 for displacement and order 1 for pressure. The Newton–Raphson algorithm (Kelley, 2003) was added for local linearization and convergence. Finally, the non-linear problem (5) was transformed into system (6) at each time step and P was called the poroelastic matrix.

$$P\delta = f \text{ with } P = \begin{bmatrix} p_1 & p_3 \\ p_4 & p_2 \end{bmatrix} \tag{6}$$

And ${}^t d$=(${}^t d^e$;${}^t d^i$)=(u^e,p^e;u^i,p^i) ${}^t f = ({}^t f^e; {}^t f^i) = (f^{u_e}, f^{p_e}; f^{u_i}, f^{p_i})$

Matrix system (6) was expressed separating internal dof δ^i and external dof δ^e. The external dof δ^e were associated to nodes of the initial FE Meshing representing a junction between the zone of interest or sub-structured domain and the complementary structures. The internal dof δ^i concerned nodes of the complementary meshing of the disc involving nucleus puplosus and annulus fibrosus.

The management of internal dof δ^i as matrix functions of external dof δ^e was the basis of the substructuring procedure. Matrix P involved four sub-matrices: p_1, p_2, p_3 and p_4. The quasi-static equilibrium was computed at each time step which allowed establishing the linear dependence of internal dof δ^i with selected external dof δ^e. Internal dof δ^i were replaced by external dof δ^e using the second line of system (6). The result was put into the first line of (6) to obtain the sub-structured poroelastic matrix $\overline{P}$ expressed by Eq. (7). The size of (7) was dependant upon the number of external dof δ^e.

$$\overline{P}\delta^e = \overline{f} \text{ with } \overline{P} = (p_1 - p_3 p_2^{-1} p_4) \tag{7}$$

and

$${}^t\delta^e = (u^e, p^e) \quad \delta^i = -p_2^{-1}p_4\delta^e + p_2^{-1}f^i \quad \overline{f} = f^e - p_3 p_2^{-1} f^i$$

When the creep response was investigated, the time-dependant computation of system (6) at time t=0 allowed computation of each term of the initial sub-structured matrix $\overline{P}_0$. This matrix combined contributions of structural matrix K, fluid matrix H and fluid-structure coupling matrix Q. At time t=+∞, influence of fluid pressure was negligible due to pressure relaxation and the viscoelastic response converged towards the quasi-static linear response. As a result, sub-structured poroelastic matrix $\overline{P}_\infty$ was obtained by suppressing the contribution of pressure dof in system (7). Between these two states, t=0 and t=+∞, the time-dependant poroelastic matrix $\overline{P}(t)$ was fitted using an exponential form with a decay factor η as expressed by Eq. (8).

$$\overline{P}(t) = \overline{P}_{+\infty} + (\overline{P}_0 - \overline{P}_{+\infty})e^{-\eta t} \tag{8}$$

Finally, the computational model was implemented in Python programming language (Python Software Foundation®) using Pysparse module to solve the spatio-temporal finite element matrix systems. Only one full computation of the initial matrix system (5) and the decay factor η were required to predict the time response of the substructure, which was beneficial in terms of computation time saving.

2.3. Application to a L_5–S_1 intervertebral disc

MRI images of the L_5–S_1 segment of a twelve-year-old male scoliotic patient were obtained using a turbo spin echo T_2-weighted MRI sequence (Magneton Vision, 1.5 T). This was achieved according to a validated clinical infant method (Violas et al., 2005). The global coordinate system was defined according to JCS (*Joint Coordinate System)* (Wu et al., 2002). Cranial part and caudal part of the disc included the outer layers of vertebral end plates. As shown in Fig. 1, contour segmentation, volume reconstruction and meshing were achieved using a custom-made image processing software developed with Python®.

Marc/Mentat (MSC software®), a finite element-based commercial code, was used to calculate the transient behavior of the intervertebral disc. The meshing was constituted by mixed brick-20 Hermann elements with 3 dof in displacement and 1 dof in pressure per node. The model involved 462 elements, 250 nodes and